

论文

BLOW-UP SOLUTIONS FOR A CLASS OF NONLINEAR PARABOLIC EQUATIONS WITH MIXED BOUNDARY CONDITIONS

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摘要 The type of problem under consideration is

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$\left\{ \begin{array}{l} \begin{aligned} & u_t = \nabla(a(u)b(x))\nabla u + g(x,q,t)f(u) & \text{in } D \times (0,T), \\ & u=0 \quad \text{on } \Gamma_1 \times (0,T), \sim \end{aligned} \\ & \frac{\partial u}{\partial n} + \sigma(x,t)u = 0 & \text{on } \Gamma_2 \times (0,T), \& \Gamma_1 \cup \Gamma_2 = \partial D, \end{array} \right.$

$u(x,0) = u_0(x) \geq 0, \not\equiv 0 \text{ in } \overline{D},$

\end{array}

$\right.$

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where D is a smooth bounded domain of R^N , $q \neq -1/\nabla u^2$. By constructing

an auxiliary function and using Hopf's maximum principles on it, existence theorems

of blow-up solutions, upper bound of "blow-up time" and upper estimates of "blow-up

rate" are given under suitable assumptions on a, b, f, g, σ and initial date

$u_0(x)$. The obtained results are applied to some examples in which a, b, f, g and

σ are power functions or exponential functions.

关键词 [Nonlinear parabolic equations, blow-up s](#)

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Abstract The type of problem under consideration is $\left\{ \begin{array}{l} \begin{aligned} & u_t = \nabla(a(u)b(x))\nabla u + g(x,q,t)f(u) & \text{in } D \times (0,T), \\ & u=0 \quad \text{on } \Gamma_1 \times (0,T), \sim \end{aligned} \\ & \frac{\partial u}{\partial n} + \sigma(x,t)u = 0 & \text{on } \Gamma_2 \times (0,T), \& \Gamma_1 \cup \Gamma_2 = \partial D, \end{array} \right.$

$u_{xx} + \sigma(x,t)u = 0$ on $\Omega \times (0,T)$, $\sigma \in L^{\frac{2N}{N+2}}(\Omega \times (0,T))$.
Initial condition: $u(x,0) = u_0(x)$, $u_0 \in C^2(\bar{\Omega})$.
Boundary condition: $u = 0$ on $\partial\Omega$.
Theorem: If $\sigma \geq 0$ and $u_0 \in C^2(\bar{\Omega})$ is non-negative, then $u \geq 0$ in $\Omega \times (0,T)$.
Proof: By contradiction, assume there exists a point $(x_0, t_0) \in \Omega \times (0, T)$ such that $u(x_0, t_0) < 0$. Let $r = \inf_{(x,t) \in \Omega \times (0,T)} u(x,t)$. Then $r < 0$. Define $v(x,t) = -u(x,t)$. Then $v(x_0, t_0) = -r > 0$. Since $u \geq 0$ on $\partial\Omega$, we have $v \geq 0$ on $\partial\Omega$. Now consider the function $w(x,t) = v(x,t) - \frac{1}{2}t^2$. Then $w(x_0, t_0) = 0$. We have $w_{tt}(x_0, t_0) = -1 < 0$. By Hopf's maximum principle, w must attain its maximum at an interior point (x_1, t_1) . At this point, we have $w_{xx}(x_1, t_1) \leq 0$ and $w_{tt}(x_1, t_1) < 0$. But since $v_{xx}(x_1, t_1) \geq 0$ (because v is a super-solution), we have $w_{xx}(x_1, t_1) \geq 0$. This contradicts the fact that w attains its maximum at (x_1, t_1) . Therefore, our assumption that $u(x_0, t_0) < 0$ is false, and we conclude that $u \geq 0$ in $\Omega \times (0,T)$.

Key words [Nonlinear parabolic equations](#) [blow-up solutions](#) [blow-up time](#) [blow-up rate](#) [maximum principles](#)

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