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The Threshold for the Erdos, Jacobson and Lehel Conjecture Being True

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摘要 Let $\sg(k,n)$ be the smallest even integer such that each n-term positive graphic sequence with term sum at least $\sg(k,n)$ can be realized by a graph containing a clique of $k+1$ vertices. Erd\H{o}s et al. (Graph Theory, 1991, 439--449) conjectured that $\sigma(k,n)=(k-1)(2n-k)+2$. Li et al. (Science in China, 1998, 510--520) proved that the conjecture is true for $k\geq 5$ and $n\geq {k\choose 2}+3$, and raised the problem of determining the smallest integer $N(k)$ such that the conjecture holds for $n\ge N(k)$. They also determined the values of $N(k)$ for $2\le k\le 7$, and proved that $\lceil\frac{5k-1}{2}\rceil \le N(k) \le {k\choose 2}+3$ for $k\ge 8$. In this paper, we determine the exact values of $\sg(k,n)$ for $n\ge 2k+3$ and $k\ge 6$. Therefore, the problem of determining $\sg(k,n)$ is completely solved. In addition, we prove as a corollary that $N(k)=\lceil\frac{5k-1}{2}\rceil$ for $k\ge 6$.

关键词 Graph Degree sequence Potentially K_{k+1}-graphic sequence

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Abstract Let $\sg(k,n)$ be the smallest even integer such that each n-term positive graphic sequence with term sum at least $\sg(k,n)$ can be realized by a graph containing a clique of $k+1$ vertices. Erd\H{o}s et al. (Graph Theory, 1991, 439--449) conjectured that $\sigma(k,n)=(k-1)(2n-k)+2$. Li et al. (Science in China, 1998, 510--520) proved that the conjecture is true for $k\geq 5$ and $n\geq {k\choose 2}+3$, and raised the problem of determining the smallest integer $N(k)$ such that the conjecture holds for $n\ge N(k)$. They also determined the values of $N(k)$ for $2\le k\le 7$, and proved that $\lceil\frac{5k-1}{2}\rceil \le N(k) \le {k\choose 2}+3$ for $k\ge 8$. In this paper, we determine the exact values of $\sg(k,n)$ for $n\ge 2k+3$ and $k\ge 6$. Therefore, the problem of determining $\sg(k,n)$ is completely solved. In addition, we prove as a corollary that $N(k)=\lceil\frac{5k-1}{2}\rceil$ for $k\ge 6$.

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