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Surgery Diagrams for Contact 3-Manifolds

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Abstract: In two previous papers, the two first-named authors introduced a notion of contact r-surgery along Legendrian knots in contact 3-manifolds. They also showed how (at least in principle) to convert any contact r-surgery into a sequence of contact (\pm 1)-surgeries, and used this to prove that any (closed) contact 3-manifold can be obtained from the standard contact structure on S^3 by a sequence of such contact (\pm 1)-surgeries. In the present paper, we give a shorter proof of that result and a more explicit algorithm for turning a contact r-surgery into (\pm 1)-surgeries. We use this to give explicit surgery diagrams for all contact structures on S^3 and $S^1 \times S^2$, as well as all overtwisted contact structures on arbitrary closed, orientable 3-manifolds. This amounts to a new proof of the Lutz-Martinet theorem that each homotopy class of 2-plane fields on such a manifold is represented by a contact structure.

Key Words: contact 3-manifold, Legendrian link, contact surgery, spin^c structure