Acta Mathematica Sinica, English Series 2007 23 (4): 735-744 ISSN: 0583-1431 CN: 11-1914/R

23(4)

Global Poincar'e inequalities on the Heisenberg group and applications

Yuxin Dong(1), 陆国震(2), Li Jing SUN(2)

(1)上海复旦大学数学系; (2)Department of Mathematics, Wayne State University, Detroit, MI 48202, USA

收稿日期 2004-6-25 修回日期 网络版发布日期 2007-3-5 接受日期 2004-9-27

摘要

关键词 Heisenberg group Sobolev inequalities Poincar\'e inequalities best constants

分类号 46E35

Global Poincar′e inequalities on the Heisenberg group and applications

Yu Xin DONG(1), Guo Zhen LU(2), Li Jing SUN(2)

(1)Department of Mathematics Fudan University Shanghai, China 200433; (2)Department of Mathematics, Wayne State University, Detroit, MI 48202 USA Department of Mathematics Wayne State University

Abstract Let f be in the localized nonisotropic Sobolev space $W_{\rm loc}^{1,p}(\Bbb H^n) $ on the n-dimensional Heisenberg group $\Bbb H^n=\Bbb C^n\times \Bbb R$, where $1\leq p <Q$ and $Q=2n+2$ is the homogeneous dimension of $\Bbb H^n$. Suppose that the subelliptic gradient is gloablly L^p integrable, i.e., $\int_{\Bbb H^n}|\nabla_{\Bbb H^n} f|^{p} du$ is finite. We prove a Poincar\'{e} inequality for f on the entire space $\Bbb H^n$. Using this inequality we prove that the function f subtracting a certain constant is in the nonisotropic Sobolev space formed by the completion of $C^\infty_{0}(\Bbb H^n)$ under the norm of $$\left(\int_{\Bbb H^n}|f|^{\frac{Qp}{Q-p}}\right)^{\frac{Q-p}{Qp}} +\left(\int_{\Bbb H^n}|\nabla_{\Bbb H^n} f|^p\right)^{\frac{1}{p}}.$$ We will also prove that the best constants and extremals for such Poincar\'e inequalities on $\Bbb H^n$ are the same as those for Sobolev inequalities on $\Bbb H^n$. Using the results of Jerison and Lee on the sharp constant and extremals for L^2 to $L^{\frac{2Q}{Q-2}}$ Sobolev inequality on the Heisenberg group, we thus arrive at the explicit best constant for the aforementioned Poincar\'e inequality on $\Bbb H^n$ when $p=2$. We also derive the lower bound of the best constants for local Poincar\'e inequalities over metric balls on the Heisenberg group $\Bbb H^n$.

Key words Heisenberg group Sobolev inequalities Poincar\'e inequalities best constants

DOI: 10.1007/s10114-005-0874-0

通讯作者 陆国震 gzlu@math.wayne.edu,gzlu2001@yahoo.com,gzlu@wayne.edu