Mathematics > Group Theory

A characterization of normal subgroups via n-closed sets

Ayman Badawi

(Submitted on 25 Jul 2011)

Let (G, *) be a semigroup, D subset of G, and n >= 2 be an integer. We say that (D, *) is an n-closed subset of G if a_1* ... *a_n in D for every a_1, ..., a_n in D. Hence every closed set is a 2-closed set. The concept of n-closed sets arise in so many natural examples. For example, let D be the set of all odd integers, then (D, +) is a 3-closed subset of (Z, +) that is not a 2-closed subset of (Z, +). If K = {1, 4, 7, 10, ...}, then (K, +) is a 4-closed subset of (Z, +) that is not an n-closed subset of (Z, +) for n = 2, 3. In this paper, we show that if (H, *) is a subgroup of a group (G, *) such that [H: G] = n < infty, then H is a normal subgroup of G if and only if every left coset of H is an (n+1)-closed subset of G.

Subjects: **Group Theory (math.GR)**
Cite as: **arXiv:1107.5058 [math.GR]**
(or **arXiv:1107.5058v1 [math.GR]** for this version)

Submission history

From: Ayman Badawi Professor [view email]
[v1] Mon, 25 Jul 2011 20:05:55 GMT (4kb)

Which authors of this paper are endorsers?