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On the Efficiency of Finite Simple Semigroups

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Abstract: Let S be a finite simple semigroup, given as a Rees matrix semigroup $\mathcal{M}[G;I,\Lambda;P]$ over a group G. We prove that the second homology of S is $H_2(S)=H_2(G)\times \mathbb{Z}^{(|I|-1)(|\Lambda|-1)}$. It is known that for any finite presentation $\langle\: A\: |\: R\: \rangle$ of S we have $|R|-|A|\geq \mbox{rank}(H_2(S))$; we say that S is efficient if equality is attained for some presentation. Given a presentation $\langle\: A_1\: |\: R_1\: \rangle$ for G, we find a presentation $\langle\: A\: |\: R\: \rangle$ for S such that $|R|-|A|=|R_1|-|A_1|+(|I|-1)(|\Lambda|-1)+1$. Further, if R_1 contains a relation of a special form, we show that $|R|-|A|$ can be reduced by one. We use this result to prove that S is efficient whenever G is finite abelian or dihedral of even degree.