Nonlinear Sciences > Exactly Solvable and Integrable Systems

Solutions of matrix NLS systems and their discretisations: A unified treatment

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Using a bidifferential graded algebra approach to integrable partial differential or difference equations, a unified treatment of continuous, semi-discrete (Ablowitz-Ladik) and fully discrete matrix NLS systems is presented. These equations originate from a universal equation within this framework, by specifying a representation of the bidifferential graded algebra and imposing a reduction. By application of a general result, corresponding families of exact solutions are obtained that in particular comprise the matrix soliton solutions in the focusing NLS case. The solutions are parametrised in terms of constant matrix data subject to a Sylvester equation (which previously appeared as a rank condition in the integrable systems literature). These data exhibit a certain redundancy, which we diminish to a large extent.
More precisely, we first consider more general AKNS-type systems from which two different matrix NLS systems emerge via reductions. In the continuous case, the familiar Hermitian conjugation reduction leads to a continuous matrix (including vector) NLS equation, but it is well-known that this does not work as well in the discrete cases. On the other hand there is a complex conjugation reduction, which apparently has not been studied previously. It leads to square matrix NLS systems, but works in all three cases (continuous, semi- and fully-discrete). A large part of this work is devoted to an exploration of the corresponding solutions, in particular regularity and asymptotic behaviour of matrix soliton solutions.

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