Applications of the quadratic covariation differentiation theory: variants of the Clark-Ocone and Stroock's formulas

Hassan Allouba, Ramiro Fontes

(Submitted on 5 Nov 2010)

In a 2006 article (\cite{A1}), Allouba gave his quadratic covariation differentiation theory for It\^o's integral calculus. He defined the derivative of a semimartingale with respect to a Brownian motion as the time derivative of their quadratic covariation and a generalization thereof. He then obtained a systematic differentiation theory containing a fundamental theorem of stochastic calculus relating this derivative to It\^o's integral, a differential stochastic chain rule, a differential stochastic mean value theorem, and other differentiation rules. Here, we use this differentiation theory to obtain variants of the Clark-Ocone and Stroock formulas, with and without change of measure. We prove our variants of the Clark-Ocone formula under L^{2}-type conditions; with no Malliavin calculus, without the use of weak distributional or Radon-Nikodym type derivatives, and without the significant machinery of the Hida-Malliavin calculus. Unlike Malliavin or Hida-Malliavin calculi, the form of our variant of the Clark-Ocone formula under change of measure is as simple as it is under no change of measure, and without requiring any further differentiability conditions on the Girsanov transform integrand beyond Novikov's condition. This is due to the invariance under change of measure of the first author's derivative in \cite{A1}. The formulations and proofs are natural applications of the differentiation theory in \cite{A1} and standard It\^o integral calculus. Iterating our Clark-Ocone formula, we obtain variants of Stroock's formula. We illustrate the applicability of these formulas by easily, and without Hida-Malliavin methods, obtaining the representation of the Brownian indicator $F=\mathbb{I}_{[K,\infty)}(W_{T})$, which is not standard Malliavin differentiable, and by applying them to digital options in finance. We then identify the chaos expansion of the Brownian indicator.

Comments: 25 pages, 3 appendices. See expanded abstract. This article gives one of several types of applications of the theory in my 2006 article (whose preprint version is arXiv:1005.4357v1)

Subjects: **Probability (math.PR)**; Classical Analysis and ODEs (math.CA); Computational Finance (q-fin.CP)

MSC classes: 60H05, 60H10, 60H99, 60G20, 60G05

Cite as: **arXiv:1011.1475v1 [math.PR]**

Submission history