Localized bases for finite dimensional homogenization approximations with non-separated scales and high-contrast

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We construct finite-dimensional approximations of solution spaces of divergence form operators with L^∞-coefficients. Our method does not rely on concepts of ergodicity or scale-separation, but on the property that the solution of space of these operators is compactly embedded in H^1 if source terms are in the unit ball of L^2 instead of the unit ball of H^{-1}. Approximation spaces are generated by solving elliptic PDEs on localized sub-domains with source terms corresponding to approximation bases for H^2. The H^1-error estimates show that $\mathcal{O}(h^{-d})$-dimensional spaces with basis elements localized to sub-domains of diameter $\mathcal{O}(h^\alpha \ln \frac{1}{h})$ (with $\alpha \in [1/2,1)$) result in an $\mathcal{O}(h^{2-2\alpha})$ accuracy for elliptic, parabolic and hyperbolic problems. For high-contrast media, the accuracy of the method is preserved provided that localized sub-domains contain buffer zones of width $\mathcal{O}(h^\alpha \ln 1/h)$ where the contrast of the medium remains bounded.
The proposed method can naturally be generalized to vectorial equations (such as elasto-dynamics).

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