

Monotone Hurwitz numbers and the HCIZ integral

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In this article, we study the topological expansion of the Harish-Chandra-Itzykson-Zuber matrix model. We prove three types of results concerning the free energy of the HCIZ model. First, at the exact level, we express each derivative of the HCIZ free energy as an absolutely convergent series in inverse powers of the ensemble dimension. The coefficients in this series are generating polynomials for a desymmetrization of the double Hurwitz numbers which we call monotone double Hurwitz numbers. Second, we prove that the genus-specific generating functions for the monotone double Hurwitz numbers are convergent power series with a common dominant singularity at the critical point $2/27$. The analytic functions defined by these series are candidate orders for a conjectural asymptotic expansion of the free energy postulated by Matytsin. Finally, we prove that under a non-vanishing hypothesis on the partition function the HCIZ free energy converges to the generating function for genus zero monotone double Hurwitz numbers uniformly on compact subsets of a complex domain.

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