



Poisson algebras, Weyl algebras and Jacobi pairs

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We study Jacobi pairs in details and obtained some properties. We also study the natural Poisson algebra structure $(\mathbb{P}, [\dots, \dots], \dots)$ on the space $\mathbb{P} = \mathbb{C}[y][[x^{-\frac{1}{N}}]]$ for some sufficient large N , and introduce some automorphisms of $(\mathbb{P}, [\dots, \dots], \dots)$ which are (possibly infinite but well-defined) products of the automorphisms of forms e^{ad_H} for $H \in x^{1-\frac{1}{N}}\mathbb{C}[y][[x^{-\frac{1}{N}}]]$ and $\tau_c: (x, y) \mapsto (x, y - cx^{-1})$ for some $c \in \mathbb{C}$. These automorphisms are used as tools to study Jacobi pairs in \mathbb{P} . In particular, starting from a Jacobi pair (F, G) in $\mathbb{C}[x, y]$ which violates the two-dimensional Jacobian conjecture, by applying some variable change $(x, y) \mapsto (x^b, x^{1-b}(y + a_1 x^{-b_1} + \dots + a_k x^{-b_k}))$ for some $b, b_i \in \mathbb{Q}_+, a_i \in \mathbb{C}$ with $b_i < 1 < b$, we obtain a \mathbb{Q} pair still denoted by (F, G) in $\mathbb{C}[x^{\frac{1}{m}}, y]$ with the form $F = x^{\frac{m}{m+n}}(f + F_0)$, $G = x^{\frac{n}{m+n}}(g + G_0)$ for some positive integers m, n , and $f, g \in \mathbb{C}[y]$, $F_0, G_0 \in x^{-\frac{1}{N}}\mathbb{C}[x^{-\frac{1}{N}}, y]$, such that F, G satisfy some additional conditions. Then we generalize the results to the Weyl algebra $W = \mathbb{C}[v][[u^{-\frac{1}{N}}]]$ with relation $[u, v] = 1$, and obtain some properties of pairs (F, G) satisfying $[F, G] = 1$, referred to as Dixmier pairs.

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