Mathematics > Algebraic Geometry

Binary hidden Markov models and varieties

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(Submitted on 3 Jun 2012 (v1), last revised 3 Sep 2012 (this version, v3))

The technological applications of hidden Markov models have been extremely diverse and successful, including natural language processing, gesture recognition, gene sequencing, and Kalman filtering of physical measurements. HMMs are highly non-linear statistical models, and just as linear models are amenable to linear algebraic techniques, non-linear models are amenable to commutative algebra and algebraic geometry.

This paper closely examines HMMs in which all the hidden random variables are binary. Its main contributions are (1) a birational parametrization for every such HMM, with an explicit inverse for recovering the hidden parameters in terms of observables, (2) a semialgebraic model membership test for every such HMM, and (3) minimal defining equations for the 4-node fully binary model, comprising 21 quadrics and 29 cubics, which were computed using Grobner bases in the cumulant coordinates of Sturmfels and Zwiernik. The new model parameters in (1) are rationally identifiable in the sense of Sullivant, Garcia-Puente, and Spielvogel, and each model's Zariski closure is therefore a rational projective variety of dimension 5. Grobner basis computations for the model and its graph are found to be considerably faster using these parameters. In the case of two hidden states, item (2) supersedes a previous algorithm of Schonhuth which is only generically defined, and the defining equations (3) yield new invariants for HMMs of all lengths ≥ 4. Such invariants have been used successfully in model selection problems in phylogenetics, and one can hope for similar applications in the case of HMMs.

Subjects:	**Algebraic Geometry (math.AG)**; Machine Learning (stat.ML)
MSC classes:	14Q15
Cite as:	**arXiv:1206.0500 [math.AG]**
	(or **arXiv:1206.0500v3 [math.AG]** for this version)

Submission history

From: Andrew Critch [view email]
[v1] Sun, 3 Jun 2012 23:43:17 GMT (30kb)
[v2] Tue, 10 Jul 2012 01:58:16 GMT (31kb)
[v3] Mon, 3 Sep 2012 04:37:33 GMT (31kb)

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