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# Metric Geometry and Collapsibility

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Cheeger's finiteness theorem bounds the number of diffeomorphism types of manifolds with bounded curvature, diameter and volume; the Hadamard--Cartan theorem, as popularized by Gromov, shows the contractibility of all non-positively curved simply connected metric length spaces. We establish a discrete version of Cheeger's theorem ("In terms of the number of facets, there are only exponentially many geometric triangulations of Riemannian manifolds with bounded geometry"), and a discrete version of the Hadamard--Cartan theorem ("Every complex that is CAT(0) with a metric for which all vertex stars are convex, is collapsible"). The first theorem has applications to discrete quantum gravity; the second shows that Forman's discrete Morse theory may be even sharper than classical Morse theory, in bounding the homology of a manifold. In fact, although Whitehead proved in 1939 that all PL collapsible manifolds are balls, we show that some collapsible manifolds are not balls.

Further central consequences of our work are:

(1) Every flag connected complex in which all links are strongly connected, is Hirsch. (This strengthens a result by Provan--Billera.)

(2) Any linear subdivision of the d-simplex collapses simplicially, after d-2 barycentric subdivisions. (This presents progress on an old question by Kirby and Lickorish.)

(3) There are exponentially many geometric triangulations of S^d. (This interpolates between the result that polytopal d-spheres are exponentially many, and the conjecture that all triangulations of S^d are exponentially many.)

(4) If a vertex-transitive simplicial complex is CAT(0) with the equilateral flat metric, then it is a simplex. (This connects metric geometry with the evasiveness conjecture.)

(5) The space of phylogenetic trees is collapsible. (This connects discrete Morse theory to mathematical biology.)

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| Comments: | 35 pages, 5 figures. Rewritten and expanded version, with several new results, especially on complexes with convex geometric realizations (Main Theorem 5 and Main Theorem 6) and vertex-transitive complexes (section 4.1). Section 3.3 has been simplified and generalized to Riemannian manifolds |
| Subjects: | **Metric Geometry (math.MG)**; Combinatorics (math.CO); Differential Geometry (math.DG) |

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