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Evidence for a conjecture of Pandharipande

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Abstract: In his paper ``Hodge integrals and degenerate contributions", Pandharipande studied the relationship between the enumerative geometry of certain 3-folds and the Gromov-Witten invariants. In some good cases, enumerative invariants (which are manifestly integers) can be expressed as a rational combination of Gromov-Witten invariants. Pandharipande speculated that the same combination of invariants should yield integers even when they do not have any enumerative significance on the 3-fold. In the case when the 3-fold is the product of a complex surface and an elliptic curve, Pandharipande has computed this combination of invariants on the 3-fold in terms of the Gromov-Witten invariants of the surface. This computation yields surprising conjectural predictions about the genus 0 and genus 1 Gromov-Witten invariants of complex surfaces. The conjecture states that certain rational combinations of the genus 0 and genus 1 Gromov-Witten invariants are always integers. Since the Gromov-Witten invariants for surfaces are often enumerative (as oppose to 3-folds), this conjecture can often also be interpreted as giving certain congruence relations among the various enumerative invariants of a surface. In this note, we state Pandharipande's conjecture and we prove it for an infinite series of classes in the case of CP^2 blown-up at 9 points. In this case, we find generating functions for the numbers appearing in the conjecture in terms of quasi-modular forms. We then prove the integrality of the numbers by proving a certain a congruence property of modular forms that is reminiscent of Ramanujan's mod 5 congruences of the partition function.

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