Mathematics > Differential Geometry

Umbilical submanifolds of $\mathbb{S}^n\times \mathbb{R}$

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We give a complete classification of umbilical submanifolds of arbitrary dimension and codimension of $\Sf^n\times \R$, extending the classification of umbilical surfaces in $\Sf^2\times \R$ by Rabah-Souam and Toubiana as well as the local description of umbilical hypersurfaces in $\Sf^n\times \R$ by Van der Veken and Vrancken. We prove that, besides small spheres in a slice, up to isometries of the ambient space they come in a two-parameter family of rotational submanifolds whose substantial codimension is either one or two and whose profile is a curve in a totally geodesic $\Sf^1\times \R$ or $\Sf^2\times \R$, respectively, the former case arising in a one-parameter family. All of them are diffeomorphic to a sphere, except for a single element that is diffeomorphic to Euclidean space. We obtain explicit parametrizations of all such submanifolds. We also study more general classes of submanifolds of $\Sf^n\times \R$ and $\Hy^n\times \R$. In particular, we give a complete description of all submanifolds in those product spaces for which the tangent component of a unit vector field spanning the factor $\R$ is an eigenvector of all shape operators. We show that surfaces with parallel mean curvature vector in $\Sf^n\times \R$ and $\Hy^n\times \R$ having this property are rotational surfaces. We also prove a Dajczer-type reduction of codimension theorem for submanifolds of $\Sf^n\times \R$ and $\Hy^n\times \R$.

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