



Mathematics > Differential Geometry

# Optimal paths for symmetric actions in the unitary group

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Given a positive and unitarily invariant Lagrangian  $L$  defined in the algebra of Hermitian matrices, and a fixed interval  $[a,b] \subset \mathbb{R}$ , we study the action defined in the Lie group of  $n \times n$  unitary matrices  $\mathcal{U}(n)$  by  $S(\alpha) = \int_a^b L(\dot{\alpha}(t)) dt$ , where  $\alpha: [a,b] \rightarrow \mathcal{U}(n)$  is a rectifiable curve. We prove that the one-parameter subgroups of  $\mathcal{U}(n)$  are the optimal paths, provided the spectrum of the exponent is bounded by  $\pi$ . Moreover, if  $L$  is strictly convex, we prove that one-parameter subgroups are the unique optimal curves joining given endpoints. Finally, we also study the connection of these results with unitarily invariant metrics in  $\mathcal{U}(n)$  as well as angular metrics in the Grassmann manifold

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