Mathematics > Differential Geometry

Problème de Plateau, équations fuchsiennes et problème de Riemann-Hilbert

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(Submitted on 4 Mar 2010)

This dissertation is devoted to the resolution of the Plateau problem in the case of a polygonal boundary in the three-dimensional euclidean space. It relies on a method developped by Ren\'e Garnier and published in 1928 in a paper which seems today to be totally forgotten. Even if Garnier's method is more geometrical and constructive than the variational one, it is sometimes really complicated, and even obscure or incomplete. We rewrite his proof with a modern formalism, we fill some gaps, and we propose some alternative easier proofs. This work mainly relies on a systematic use of Fuchsian systems and on the relation that we establish between the reality of such systems and their monodromy. Garnier's method is based on the following result: using the spinorial Weierstrass representation for minimal surfaces, we can associate to each minimal disk with a polygonal boundary a real Fuchsian second order equation. The monodromy of the equation is encoded by the directions of the edges of the boundary. To solve the Plateau problem, we are thus led to solve a Riemann-Hilbert problem. Then, we proceed in two steps: first, by means of isomonodromic deformations, we construct the familly of all minimal disks with a polygonal boundary with given directions. Then, by studying the edges's lengths of these polygonal boundaries, we show that every polygon is the boundary of a minimal disk.

Comments:	122 pages
Subjects:	**Differential Geometry (math.DG)**; Exactly Solvable and Integrable Systems (nlin.SI)
MSC classes:	53A10, 34M03, 34M35, 34M50, 34M55, 34M56
Cite as:	**arXiv:1003.0978v1 [math.DG]**

Submission history

From: Laura Desideri [view email] [via CCSD proxy]
[v1] Thu, 4 Mar 2010 07:26:39 GMT (245kb)

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