



# Hairy graphs and the unstable homology of $\text{Mod}(g,s)$ , $\text{Out}(F_n)$ and $\text{Aut}(F_n)$

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We study a family of Lie algebras  $\{hO\}$  which are defined for cyclic operads  $O$ . Using his graph homology theory, Kontsevich identified the homology of two of these Lie algebras (corresponding to the Lie and associative operads) with the cohomology of outer automorphism groups of free groups and mapping class groups of punctured surfaces, respectively. In this paper we introduce a hairy graph homology theory for  $O$ . We show that the homology of  $hO$  embeds in hairy graph homology via a trace map which generalizes the trace map defined by S. Morita. For the Lie operad we use the trace map to find large new summands of the abelianization of  $hO$  which are related to classical modular forms for  $SL(2,Z)$ . Using cusp forms we construct new cycles for the unstable homology of  $\text{Out}(F_n)$ , and using Eisenstein series we find new cycles for  $\text{Aut}(F_n)$ . For the associative operad we compute the first homology of the hairy graph complex by adapting an argument of Morita, Sakasai and Suzuki, who determined the complete abelianization of  $hO$  in the associative case.

Comments: Some typos fixed. In an earlier version, we had made a conjecture about the image of the trace map, which we have proven and will be included in a future paper. Some comments in this paper have been changed to reflect this. To appear in J. Top

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