Mathematics > Algebraic Topology

Combinatorial realisation of cycles and small covers

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In 1940s Steenrod asked if every homology class $z\in H_n(X,\mathbb{Z})$ of every topological space X can be realised by an image of the fundamental class of an oriented closed smooth manifold. Thom found a non-realisable 7-dimensional class and proved that for every n, there is a positive integer $k(n)$ such that the class $k(n)z$ is always realisable. The proof was by methods of algebraic topology and gave no information on the topology the manifold which realises the homology class. We give a purely combinatorial construction of a manifold that realises a multiple of a given homology class. For every n, this construction yields a manifold M^n_0 with the following universality property: For any X and $z\in H_n(X,\mathbb{Z})$, a multiple of z can be realised by an image of a (non-ramified) finite-sheeted covering of M^n_0. Manifolds satisfying this property are called URC-manifolds. The manifold M^n_0 is a so-called small cover of the permutahedron, i.e., a manifold glued in a special way out of 2^n permutahedra. (The permutahedron is a special convex polytope with $(n+1)!$ vertices.) Among small covers over other simple polytopes, we find a broad class of examples of URC-manifolds. In particular, in dimension 4, we find a hyperbolic URC-manifold. Thus we obtain that a multiple of every homology class can be realised by an image of a hyperbolic manifold, which was conjectured by Kotschick and L\"oh. Finally, we investigate the relationship between URC-manifolds and simplicial volume.

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