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Asymptotics for the Korteweg-de Vries-Burgers Equation

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摘要 We study large time asymptotics of solutions to the Korteweg--de Vries--Burgers equation $$ u_{t}+uu_{x}-u_{xx}+u_{xxx}=0,\text{ }x\in \mathbf{R,}\text{ }t>0. $$ \no We are interested in the large time asymptotics for the case when the initial data have an arbitrary size. We prove that if the initial data $% u_{0}\in \mathbf{H}^{s}\left(\mathbf{R}\right) \cap \mathbf{L}^{1}\left(\mathbf{R}\right) ,$ where $s>-\frac{1}{2}$, then there exists a unique solution $u\left(t,x\right) \in \mathbf{C}^{\infty }\left(\left(0,\infty \right) ;\mathbf{H}^{\infty }\left(\mathbf{R}\right) \right) $ to the Cauchy problem for the Korteweg--de Vries--Burgers equation, which has asymptotics \begin{equation*} u\left(t\right) =t^{-\frac{1}{2}}f_{M}((\cdot) t^{-\frac{% 1}{2}}) +o(t^{-\frac{1}{2}}) \end{equation*}% \no as $t\rightarrow \infty ,$ where f_{M} is the self-similar solution for the Burgers equation. Moreover if $xu_{0}\left(x\right) \in \mathbf{L}% ^{1}\left(\mathbf{R}\right),$ then the asymptotics are true \begin{equation*} u\left(t\right) =t^{-\frac{1}{2}}f_{M}((\cdot) t^{-\frac{% 1}{2}}) +O(t^{-\frac{1}{2}-\gamma }), \end{equation*}% \no where $\gamma \in \left(0,\frac{1}{2}\right)$.

关键词 Korteweg--de Vries--Burgers equation asymptotics for large time large initial data

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Abstract We study large time asymptotics of solutions to the Korteweg--de Vries--Burgers equation $$ u_{t}+uu_{x}-u_{xx}+u_{xxx}=0,\text{ }x\in \mathbf{R,}\text{ }t>0. $$ \no We are interested in the large time asymptotics for the case when the initial data have an arbitrary size. We prove that if the initial data $% u_{0}\in \mathbf{H}^{s}\left(\mathbf{R}\right) \cap \mathbf{L}^{1}\left(\mathbf{R}\right) ,$ where $s>-\frac{1}{2}$, then there exists a unique solution $u\left(t,x\right) \in \mathbf{C}^{\infty }\left(\left(0,\infty \right) ;\mathbf{H}^{\infty }\left(\mathbf{R}\right) \right) $ to the Cauchy problem for the Korteweg--de Vries--Burgers equation, which has asymptotics \begin{equation*} u\left(t\right) =t^{-\frac{1}{2}}f_{M}((\cdot) t^{-\frac{% 1}{2}}) +o(t^{-\frac{1}{2}}) \end{equation*}% \no as $t\rightarrow \infty ,$ where f_{M} is the self-similar solution for the Burgers equation. Moreover if $xu_{0}\left(x\right) \in \mathbf{L}% ^{1}\left(\mathbf{R}\right),$ then the asymptotics are true \begin{equation*} u\left(t\right) =t^{-\frac{1}{2}}f_{M}((\cdot) t^{-\frac{% 1}{2}}) +O(t^{-\frac{1}{2}-\gamma }), \end{equation*}% \no where $\gamma \in \left(0,\frac{1}{2}\right)$.

Key words Korteweg--de Vries--Burgers equation asymptotics for large time large initial data

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