Piecewise Affine Selections for Piecewise Polyhedral Multifunctions and Metric Projections

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Abstract: Piecewise polyhedral multifunctions are the set-valued version of piecewise affine functions. We investigate selections of piecewise polyhedral multifunctions, in particular, the least norm selection and continuous extremal point selections.

A special class of piecewise polyhedral multifunctions is the collection of metric projections $\Pi_{K,P}$ from $\mathbb{R}^n$ (endowed with a polyhedral norm $\|\cdot\|_P$) to a polyhedral subset K of $\mathbb{R}^n$. As a consequence, the two types of selections are piecewise affine selections for $\Pi_{K,P}$. Moreover, if $\Pi_{K,\infty}$ and $\Pi_{K,1}$ are the metric projection onto K in $\mathbb{R}^n$ endowed with the ℓ_∞-norm and the ℓ_1-norm, respectively, we prove that $\Pi_{K,1}$ has a piecewise affine and quasi-linear extremal point selection when K is a subspace, and that the strict best approximation $\operatorname{\mathbf{sba}}_K(x)$ of x in K is a piecewise affine selection for $\Pi_{K,\infty}$.

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