

# Euler-Lagrange Inclusions and Existence of Minimizers for a Class of Non-Coercive Variational Problems

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**Abstract:** We are concerned with integral functionals of the form  $J(v) = \int_{B_R^n} \left( f(|x|, |\nabla v(x)|) + h(|x|, v(x)) \right) dx$ , defined on  $W^{1,1}_0(B_R^n, \mathbb{R}^m)$ , where  $B_R^n$  is the ball of  $\mathbb{R}^n$  centered at the origin and with radius  $R > 0$ . We assume that the functional  $J$  is convex, but the compactness of the sublevels of  $J$  is not required. We prove that, under suitable assumptions on  $f$  and  $h$ , there exists a radially symmetric minimizer  $v \in W^{1,1}_0(B_R, \mathbb{R}^m)$  for  $J$ . Moreover, we associate to the functional  $J$  a system of differential inclusions of the Euler-Lagrange type, and we prove that the solvability of these inclusions is a necessary and sufficient condition for the existence of a radially symmetric minimizer for  $J$ .

**Keywords:** Calculus of variations, existence, Euler-Lagrange inclusions, radially symmetric solutions, non-coercive problems

**Classification (MSC2000):** 49J10, 49K05; 49J30

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