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On a Non-Standard Convex Regularization and the Relaxation of Unbounded Integral Functionals of the Calculus of Variations

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Abstract: The analysis of the relationships between the functional $F^{(\infty)}(\Omega,\cdot) \colon u \in W^{1,\infty}(\Omega) \mapsto \inf$ $\{\liminf_h \int_\Omega f(\nabla u_h)dx : \{u_h\}$ $\subseteq W^{1,\infty}(\Omega), > u_h \to u$ in weak*-$W^{1,\infty}(\Omega)\}$, and the sequential weak*-$W^{1,\infty}(\Omega)$-relaxed functional ${\overline F}^{(\infty)}(\Omega,\cdot)$ of the integral $u \in W^{1,\infty}(\Omega) \mapsto \int_\Omega f(\nabla u)dx$ is carried out, where $f\colon \mathbb{R}^n \to [0,+\infty]$, Ω is a bounded open subset of $\mathbb{R}^n$, and $u\in W^{1,\infty}(\Omega)$.
In [8] it has been proved the existence of $f^{(\infty)}\colon \mathbb{R}^n \to [0,+\infty]$ such that $F^{(\infty)}(\Omega,u) = \int_\Omega f^{(\infty)}(\nabla u)dx$ for every convex bounded open set Ω, $u \in W^{1,\infty}(\Omega)$ such that $F^{(\infty)}(\Omega,u) < +\infty$, and this result is exploited there to deduce that ${\overline F}^{(\infty)}(\Omega,u)=\int_\Omega f^{\ast\ast}(\nabla u)dx$ for every convex bounded open set Ω, $u \in W^{1,\infty}(\Omega)$, where $f^{\ast\ast}$ is the bipolar of f.
In the present paper it is first proved that $f^{(\infty)}$ is the convex envelope of the lower semicontinuous envelope of f, and an example is produced showing that $f^{(\infty)}$ may be different from $f^{\ast\ast}$. Conditions for their identity are then furnished.

Examples and conditions concerning the coincidence between $F^{(\infty)}(\Omega,u)$ and $\int_\Omega f^{(\infty)}(\nabla u)dx$ for every convex bounded open set Ω, $u\in W^{1,\infty}(\Omega)$ are also proposed.
By such results conditions for the identity between $F^{(\infty)}$ and ${\overline F}^{(\infty)}$ are deduced.

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