BV Functions with Respect to a Measure and Relaxation of Metric Integral Functionals

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Abstract: We introduce and study the space of bounded variation functions with respect to a Radon measure μ on $\mathbb{R}^N$ and to a metric integrand φ on the tangent bundle to μ. We show that it is equivalent to view such space as the class of μ-integrable functions for which a distributional notion of (μ, φ)-total variation is finite, or as the finiteness domain of a relaxed functional. We prove a quite general coarea-type formula and then we focus our attention to the problem of finding an integral representation for the (μ, φ)-total variation.

Keywords: Bounded variation functions, Radon measures, Relaxation, Duality, Integral representation

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