

# Unbounded Linear Monotone Operators on Nonreflexive Banach Spaces

R. R. Phelps and S. Simons

Department of Mathematics, Box 354350, University of Washington, Seattle, WA 98195, USA, [phelps@math.washington.edu](mailto:phelps@math.washington.edu), and Department of Mathematics, University of California, Santa Barbara, CA 93106-3080, USA, [simons@math.ucsb.edu](mailto:simons@math.ucsb.edu)

---



**Abstract:** This study of unbounded linear monotone operators  $T: E \rightarrow E^*$  for nonreflexive Banach spaces  $E$  was motivated by the (still open) problem of distinguishing between several well-studied classes of maximal monotone set-valued operators (classes which coincide when  $E$  is reflexive). It is shown in Theorem 6.7 that in the unbounded linear case these classes are closely related. They may even be identical, as was shown by Bauschke and Borwein to be true for the case of bounded linear monotone operators. (A short new proof of this latter result is given in Section 8.) Earlier sections yield a characterization of maximality, a characterization of maximal monotone unbounded linear symmetric operators (in terms of the convex function  $\|Tx, x\|$ ) and a number of relevant examples. Section 7 contains a proof that, in the linear case, Rockafellar's theorem on the maximality of the sum of two maximal monotone operators is true even in nonreflexive Banach spaces. It also contains a counterexample in Hilbert space showing that the hypotheses cannot be substantially weakened.

**Full text of the article:**

- [Compressed PostScript file](#) (122 kilobytes)
- [PDF file](#) (263 kilobytes)

---

[\[Previous Article\]](#) [\[Next Article\]](#) [\[Contents of this Number\]](#)

---