# Canonical and grand canonical partition functions of Dyson gases as tau-functions of integrable hierarchies and their fermionic realization

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The partition function for a canonical ensemble of 2D Coulomb charges in a background potential (the Dyson gas) is realized as a vacuum expectation value of a group-like element constructed in terms of free fermionic operators. This representation provides an explicit identification of the partition function with a tau-function of the 2D Toda lattice hierarchy. Its dispersionless (quasiclassical) limit yields the tau-function for analytic curves encoding the integrable structure of the inverse potential problem and parametric conformal maps. A similar fermionic realization of partition functions for grand canonical ensembles of 2D Coulomb charges in the presence of an ideal conductor is also suggested. Their representation as Fredholm determinants is given and their relation to integrable hierarchies, growth problems and conformal maps is discussed.

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