Mathematics > Optimization and Control

Geodesic boundary value problems with symmetry

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This paper shows how left and right actions of Lie groups on a manifold may be used to complement one another in a variational reformulation of optimal control problems equivalently as geodesic boundary value problems with symmetry. We prove an equivalence theorem to this effect and illustrate it with several examples. In finite-dimensions, we discuss geodesic flows on the Lie groups SO(3) and SE(3) under the left and right actions of their respective Lie algebras. In an infinite-dimensional example, we discuss optimal large-deformation matching of one closed curve to another embedded in the same plane. In the curve-matching example, the manifold $\Emb(S^1, \mathbb{R}^2)$ comprises the space of closed curves S^1 embedded in the plane $\mathbb{R}^2$. The diffeomorphic left action $\Diff(\mathbb{R}^2)$ deforms the curve by a smooth invertible time-dependent transformation of the coordinate system in which it is embedded, while leaving the parameterisation of the curve invariant. The diffeomorphic right action $\Diff(S^1)$ corresponds to a smooth invertible reparameterisation of the S^1 domain coordinates of the curve. As we show, this right action unlocks an important degree of freedom for geodesically matching the curve shapes using an equivalent fixed boundary value problem, without being constrained to match corresponding points along the template and target curves at the endpoint in time.

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