Mathematics > Analysis of PDEs

Concerning the L^4 norms of typical eigenfunctions on compact surfaces

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(Submitted on 31 Oct 2010)

Let (M,g) be a two-dimensional compact boundaryless Riemannian manifold with Laplacian, Δ_g. If e_λ are the associated eigenfunctions of $\sqrt{-\Delta_g}$ so that $-\Delta_g e_\lambda = \lambda^2 e_\lambda$, then it has been known for some time \cite {soggeest} that $\|e_\lambda\|_{L^4(M)}\lesssim \lambda^{1/8}$, assuming that e_λ is normalized to have L^2-norm one. This result is sharp in the sense that it cannot be improved on the standard sphere because of highest weight spherical harmonics of degree k. On the other hand, we shall show that the average L^4 norm of the standard basis for the space ${\mathcal H}_k$ of spherical harmonics of degree k on S^2 merely grows like $(\log k)^{1/4}$. We also sketch a proof that the average of $\sum_{j = 1}^{2k + 1} \|e_\lambda\|_{L^4}^4 $ for a random orthonormal basis of ${\mathcal H}_k$ is O(1). We are not able to determine the maximum of this quantity over all orthonormal bases of ${\mathcal H}_k$ or for orthonormal bases of eigenfunctions on other Riemannian manifolds. However, under the assumption that the periodic geodesics in (M,g) are of measure zero, we are able to show that for {\it any} orthonormal basis of eigenfunctions we have that $\|e_{\lambda_{j_k}}\|_{L^4(M)}=o(\lambda_{j_k}^{1/8})$ for a density one subsequence of eigenvalues λ_{j_k}. This assumption is generic and it is the one in the Duistermaat-Gullemin theorem \cite{dg} which gave related improvements for the error term in the sharp Weyl theorem. The proof of our result uses a recent estimate of the first author \cite{Sokakeya} that gives a necessary and sufficient condition that $\|e_\lambda\|_{L^4(M)}=o(\lambda^{1/8})$.

Comments:	15 pages
Subjects:	**Analysis of PDEs (math.AP)**
MSC classes:	Primary, 35F99, Secondary 35L20, 42C99
Cite as:	**arXiv:1011.0215v1 [math.AP]**

Submission history

From: Christopher D. Sogge [view email]
[v1] Sun, 31 Oct 2010 22:55:44 GMT (18kb)