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On the meaning of the Vakhitov-Kolokolov stability criterion for the nonlinear Dirac equation

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We consider the spectral stability of solitary wave solutions \phi(x)e^{-i\omega t} to the nonlinear Dirac equation in any dimension. This equation is well-known to theoretical physicists as the Soler model (or, in one dimension, the Gross-Neveu model), and attracted much attention for many years. We show that, generically, at the values of where the Vakhitov-Kolokolov stability criterion breaks down, a pair of real eigenvalues (one positive, one negative) appears from the origin, leading to the linear instability of corresponding solitary waves.

As an auxiliary result, we state the virial identities ("Pohozhaev theorem") for the nonlinear Dirac equation.

We also show that \pm 2\omega i are the eigenvalues of the nonlinear Dirac equation linearized at \phi(x)e^{-i\omega t}, which are embedded into the continuous spectrum for |\omega| > m/3. This result holds for the nonlinear Dirac equation with any nonlinearity of the Soler form ("scalar-scalar interaction") and in any dimension.

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