Mathematics > Spectral Theory

Estimates on Neumann eigenfunctions at the boundary, and the "Method of Particular Solutions" for computing them

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We consider the "Method of particular solutions" for numerically computing eigenvalues and eigenfunctions of the Laplacian Δ on a smooth, bounded domain Omega in RR^n with either Dirichlet or Neumann boundary conditions. This method constructs approximate eigenvalues E, and approximate eigenfunctions u that satisfy $\Delta u=Eu$ in Omega, but not the exact boundary condition. An inclusion bound is then an estimate on the distance of E from the actual spectrum of the Laplacian, in terms of (boundary data of) u. We prove operator norm estimates on certain operators on $L^2 (\partial \Omega)$ constructed from the boundary values of the true eigenfunctions, and show that these estimates lead to sharp inclusion bounds in the sense that their scaling with E is optimal. This is advantageous for the accurate computation of large eigenvalues. The Dirichlet case can be treated using elementary arguments and has appeared in SIAM J. Num. Anal. 49 (2011), 1046-1063, while the Neumann case seems to require much more sophisticated technology. We include preliminary numerical examples for the Neumann case.

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