Singular limit and exact decay rate of a nonlinear elliptic equation

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For any $n\ge 3$, $0<m\le (n-2)/n$, and constants $\eta>0$, $\beta>0$, α, satisfying $\alpha\le\beta(n-2)/m$, we prove the existence of radially symmetric solution of $\frac{n-1}{m}\Delta v^m+\alpha v +\beta x\cdot\nabla v=0$, $v>0$, in $\R^n$, $v(0)=\eta$, without using the phase plane method. When $0<m<(n-2)/n$, $n\ge 3$, and $\alpha=2\beta/(1-m)>0$, we prove that the radially symmetric solution v of the above elliptic equation satisfies $\lim_{|x|\to\infty}\frac{|x|^2v(x)^{1-m}}{\log |x|} =\frac{2(n-1)(n-2-nm)}{\beta(1-m)}$. In particular when $m=\frac{n-2}{n+2}$, $n\ge 3$, and $\alpha=2\beta/(1-m)>0$, the metric $g_{ij}=v^{\frac{4}{n+2}}dx^2$ is the steady soliton solution of the Yamabe flow on $\R^n$ and we obtain $\lim_{|x|\to\infty}\frac{|x|^2v(x)^{1-m}}{\log |x|}=\frac{(n-1)(n-2)}{\beta}$. When $0<m<(n-2)/n$, $n\ge 3$, and $2\beta/(1-m)>\max (\alpha,0)$, we prove that $\lim_{|x|\to\infty}|x|^{\alpha/\beta}v(x)=A$ for some constant $A>0$. For $\beta>0$ or $\alpha=0$, we prove that the radially symmetric solution $v^{(m)}$ of the above elliptic elliptic equation converges uniformly on every compact subset of $\R^n$ to the solution u of the equation $(n-1)\Delta\log u+\alpha u+\beta x\cdot\nabla u=0$, $u>0$, in $\R^n$, $u(0)=\eta$, as $m\to 0$.

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