Mathematics > Analysis of PDEs

Kac's Program in Kinetic Theory

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This paper is devoted to the study of propagation of chaos and mean-field limits for systems of indistinguable particles, undergoing collision processes. The prime examples we will consider are the many-particle jump processes of Kac and McKean \cite{Kac1956,McKean1967} giving rise to the Boltzmann equation. We solve the conjecture raised by Kac \cite{Kac1956}, motivating his program, on the rigorous connection between the long-time behavior of a collisional many-particle system and the one of its mean-field limit, for bounded as well as unbounded collision rates. Motivated by the inspirative paper by Gr\"unbaum \cite{Grunbaum}, we develop an abstract method that reduces the question of propagation of chaos to that of proving a purely functional estimate on generator operators ({\em consistency estimates}), along with differentiability estimates on the flow of the nonlinear limit equation ({\em stability estimates}). This allows us to exploit dissipativity at the level of the mean-field limit equation rather than the level of the particle system (as proposed by Kac). Using this method we show: (1) Quantitative estimates, that are uniform in time, on the chaoticity of a family of states. (2) Propagation of {\it entropic chaoticity}, as defined in \cite {CCLLV}. (3) Estimates on the time of relaxation to equilibrium, that are \emph{independent of the number of particles in the system}. Our results cover the two main Boltzmann physical collision processes with unbounded collision rates: hard spheres and \emph{true} Maxwell molecules interactions. The proof of the \emph{stability estimates} for these models requires significant analytic efforts and new estimates.

Comments:	101 pages. This preprint supersedes hal-00447988 / arXiv:1001.2994. It entirely includes these previous results and contains new results (relaxation rates independent of the number of particles, propagation of entropic chaos, comparison with approaches based on the BBGKY hierarchy) and some additional explanations and historical notes. This revised version corrects many typos and improves the presentation
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