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On the Hénon-Lane-Emden conjecture

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We consider Liouville-type theorems for the following H\'{e}non-Lane-Emden system
\hfill -\Delta u&=& |x|^{a}v^p \text{in} \mathbb{R}^N,
\hfill -\Delta v&=& |x|^{b}u^q \text{in} \mathbb{R}^N, when $pq>1$, $p,q,a,b\ge0$. The main conjecture states that there is no non-trivial non-negative solution whenever (p,q) is under the critical Sobolev hyperbola, i.e. $ \frac{N+a}{p+1}+\frac{N+b}{q+1}>{N-2}$.
We show that this is indeed the case in dimension N=3 provided the solution is also assumed to be bounded, extending a result established recently by Phan-Souplet in the scalar case.
Assuming stability of the solutions, we could then prove Liouville-type theorems in higher dimensions.

For the scalar cases, albeit of second order ($a=b$ and $p=q$) or of fourth order ($a\ge 0=b$ and $p>1=q$), we show that for all dimensions $N\ge 3$ in the first case (resp., $N\ge 5$ in the second case), there is no positive solution with a finite Morse index, whenever p is below the corresponding critical exponent, i.e $ 1<p<\frac{N+2+2a}{N-2}$ (resp., $ 1<p<\frac{N+4+2a}{N-4}$). Finally, we show that non-negative stable solutions of the full H\'{e}non-Lane-Emden system are trivial provided \label {sysdim00}
N<2+2(\frac{p(b+2)+a+2}{pq-1}) (\sqrt{\frac{pq(q+1)}{p+1}}+ \sqrt{\frac{pq(q+1)}{p+1}-\sqrt\frac{pq(q+1)}{p+1}}).

Comments:	Theorem 4 has been added in the new version. 23 pages, Comments are welcome. Updated version - if any - can be downloaded at this http URL or this http URL
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