



# The frequency-dependent Wright-Fisher model: diffusive and non-diffusive approximations

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We study a class of processes that are akin to the Wright-Fisher model, with transition probabilities weighted in terms of the frequency-dependent fitness of the population types. By considering an approximate weak formulation of the discrete problem, we are able to derive a corresponding continuous weak formulation for the probability density. Therefore, we obtain a family of partial differential equations (PDE) for the evolution of the probability density, and which will be an approximation of the discrete process in the joint large population, small time-steps and weak selection limit. If the fitness functions are sufficiently regular, we can recast the weak formulation in a more standard formulation, without any boundary conditions, but supplemented by a number of conservation laws. The equations in this family can be purely diffusive, purely hyperbolic or of convection-diffusion type, with frequency dependent convection. The particular outcome will depend on the assumed scalings. The diffusive equations are of the degenerate type; using a duality approach, we also obtain a frequency dependent version of the Kimura equation without any further assumptions. We also show that the convective approximation is related to the replicator dynamics and provide some estimate of how accurate is the convective approximation, with respect to the convective-diffusion approximation. In particular, we show that the mode, but not the expected value, of the probability distribution is modelled by the replicator dynamics. Some numerical simulations that illustrate the results are also presented.

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