

论文

Heisenberg群上无穷远处的集中列紧原理和具有Sobolev临界指数的 p -次Laplace方程多解的存在性

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摘要:

通过建立Heisenberg群上无穷远处的集中列紧原理, 研究了如下 p -次Laplace方程

$$-\Delta_{H,p} u = \lambda g(\xi) |u|^{q-2} u + f(\xi) |u|^{p^*-2} u, \text{ 在 } H^n \text{ 上,}$$

$$u \in D^{1,p}(H^n),$$

其中 $\xi \in H^n, \lambda \in \mathbb{R}, 1 < p < Q = 2n + 2, n \geq 1, 1 < q < p, p^* = Qp / (Q - p), g(\xi), f(\xi)$ 是可以变号和满足一定条件的函数. 在适当条件下利用集中列紧原理证明在某个水平处的Palais-Smale条件, 从而结合变分原理得到方程存在 $m-j$ 对解, 其中 $m > j$, 且 m, j 为整数.

关键词: Heisenberg 群 p -次Laplace算子 集中列紧原理 Palais-Smale条件 多解

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A Concentration-Compactness Principle at Infinity on the Heisenberg Group and Multiplicity of Solutions for p -sub-Laplacian Problem Involving Critical Sobolev Exponents

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Abstract:

The main results of this paper establish the concentration-compactness principle at infinity on the Heisenberg group. The authors consider the p -sub-Laplacian problem involving critical Sobolev exponents

$$-\Delta_{H,p} u = \lambda g(\xi) |u|^{q-2} u + f(\xi) |u|^{p^*-2} u, \text{ in } H^n,$$

$$u \in D^{1,p}(H^n),$$

where $\xi \in H^n, \lambda \in \mathbb{R}, 1 < p < Q = 2n + 2, n \geq 1, 1 < q < p, p^* = Qp / (Q - p), g(\xi)$ and $f(\xi)$ change sign and satisfy some suitable conditions. Under certain assumptions, they show the existence of $m-j$ pairs of nontrivial solutions via variational method, where $m > j$, both m and j are integers. The concentration-compactness principle allows to prove the Palais-Smale condition is satisfied below a certain level.

Keywords: Heisenberg group p -sub-Laplacian Concentration-compactness principle Palais-Smale condition Multiplicity

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