Mathematical Physics

Transitions in active rotator systems: invariant hyperbolic manifold approach

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(Submitted on 3 Jun 2011 (v1), last revised 13 Jul 2011 (this version, v2))

Our main focus is on a general class of active rotators with mean field interactions, that is globally coupled large families of dynamical systems on the unit circle with non-trivial stochastic dynamics. Each isolated system is a diffusion process on a circle, with drift -delta V', where V' is a periodic function and delta is an intensity parameter. It is well known that the interacting dynamics is accurately described, in the limit of infinitely many interacting components, by a Fokker-Planck PDE and the model reduces for delta=0 to a particular case of the Kuramoto synchronization model, for which one can show the existence of a stable normally hyperbolic manifold of stationary solutions for the corresponding Fokker-Planck equation (we are interested in the case in which this manifold is non-trivial, that happens when the interaction is sufficiently strong, that is in the synchronized regime of the Kuramoto model). We use the robustness of normally hyperbolic structures to infer qualitative and quantitative results on the |delta|< delta0 cases, with delta0 a suitable threshold: as a matter of fact, we obtain an accurate description of the dynamics on the invariant manifold for delta=0 and we link it explicitly to the potential V . This approach allows to have a complete description of the phase diagram of the active rotators model, at least for |delta|< delta0, thus identifying for which values of the parameters (notably, noise intensity and/or coupling strength) the system exhibits periodic pulse waves or stabilizes at a quiescent resting state. Moreover, some of our results are very explicit and this brings a new insight into the combined effect of active rotator dynamics, noise and interaction. The links with the literature on specific systems, notably neuronal models, are discussed in detail.

Comments:	29 pages, 4 figures. Version 2: some changes in introduction, added references
Subjects:	**Mathematical Physics (math-ph)**; Chaotic Dynamics (nlin.CD); Neurons and Cognition (q-bio.NC)
MSC classes:	37N25, 82C26, 82C31, 92B20
Cite as:	**arXiv:1106.0758 [math-ph]**
	(or **arXiv:1106.0758v2 [math-ph]** for this version)

Submission history

From: Giambattista Giacomin [view email]
[v1] Fri, 3 Jun 2011 20:16:52 GMT (50kb)
[v2] Wed, 13 Jul 2011 16:47:22 GMT (52kb)