Mathematics > Combinatorics

KP solitons and total positivity for the Grassmannian

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Soliton solutions of the KP equation have been studied since 1970, when Kadomtsev and Petviashvili proposed a two-dimensional dispersive wave equation now known as the KP equation. It is well-known that one can use the Wronskian method to construct a soliton solution to the KP equation from each point of the real Grassmannian Gr_kn. More recently several authors have studied the regular solutions that one obtains in this way: these come from points of the totally non-negative part of the Grassmannian (Gr_kn)_{>= 0}. In this paper we exhibit a surprising connection between the theory of total positivity for the Grassmannian, and the structure of regular soliton solutions to the KP equation. By exploiting this connection, we obtain new insights into the structure of KP solitons, as well as new interpretations of the combinatorial objects indexing cells of (Gr_kn)_{>= 0}. In particular, we completely classify the spatial patterns of the soliton solutions coming from (Gr_2n)_{>0}, as well as those coming from (Gr_kn)_{>= 0} when the absolute value of the time parameter is sufficiently large. We also demonstrate an intriguing connection between soliton graphs for (Gr_kn)_{>0} and the cluster algebras of Fomin and Zelevinsky. Finally, we use this connection to solve the inverse problem for generic KP solitons coming from (Gr_kn)_{>0}.

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Which authors of this paper are endorsers?