Differential equation approximations of stochastic network processes: an operator semigroup approach

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The rigorous linking of exact stochastic models to mean-field approximations is studied. Starting from the differential equation point of view the stochastic model is identified by its Kolmogorov equations, which is a system of linear ODEs that depends on the state space size (N) and can be written as $\dot u_N=A_N u_N$. Our results rely on the convergence of the transition matrices A_N to an operator A. This convergence also implies that the solutions u_N converge to the solution u of $\dot u=Au$. The limiting ODE can be easily used to derive simpler mean-field-type models such that the moments of the stochastic process will converge uniformly to the solution of appropriately chosen mean-field equations. A bi-product of this method is the proof that the rate of convergence is $\mathcal{O}(1/N)$. In addition, it turns out that the proof holds for cases that are slightly more general than the usual density dependent one. Moreover, for Markov chains where the transition rates satisfy some sign conditions, a new approach for proving convergence to the mean-field limit is proposed. The starting point in this case is the derivation of a countable system of ordinary differential equations for all the moments. This is followed by the proof of a perturbation theorem for this infinite system, which in turn leads to an estimate for the difference between the moments and the corresponding quantities derived from the solution of the mean-field ODE.

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