Mathematics > Functional Analysis

Borel structure of the spectrum of a closed operator

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For a linear operator T in a Banach space let $\sigma_p(T)$ denote the point spectrum of T, $\sigma_{p[n]}(T)$ for finite $n > 0$ be the set of all $\lambda \in \sigma_p(T)$ such that $\dim \ker (T - \lambda) = n$ and let $\sigma_{p[\infty]}(T)$ be the set of all $\lambda \in \sigma_p(T)$ for which $\ker (T - \lambda)$ is infinite-dimensional. It is shown that $\sigma_p(T)$ is $\mathcal{F}_{\sigma}$, $\sigma_{p[\infty]}(T)$ is $\mathcal{F}_{\sigma\delta}$ and for each finite n the set $\sigma_{p[n]}(T)$ is the intersection of an $\mathcal{F}_{\sigma}$ and a $\mathcal{G}_{\delta}$ set provided T is closable and the domain of T is separable and weakly σ-compact. For closed densely defined operators in a separable Hilbert space $\mathcal{H}$ more detailed decomposition of the spectra is done and the algebra of all bounded linear operators on $\mathcal{H}$ is decomposed into Borel parts. In particular, it is shown that the set of all closed range operators on $\mathcal{H}$ is Borel.

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