Wavelet analysis on adeles and pseudo-differential operators

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This paper is devoted to wavelet analysis on adele ring $\bA$ and the theory of pseudo-differential operators. We develop the technique which gives the possibility to generalize finite-dimensional results of wavelet analysis to the case of adeles $\bA$ by using infinite tensor products of Hilbert spaces. The adele ring is roughly speaking a subring of the direct product of all possible (p-adic and Archimedean) completions $\bQ_p$ of the field of rational numbers $\bQ$ with some conditions at infinity. Using our technique, we prove that $L^2(\bA)=\otimes_{e,p\in\{\infty,2,3,5,...}}L^2({\bQ}_{p})$ is the infinite tensor product of the spaces $L^2({\bQ}_{p})$ with a stabilization $e=(e_p)_p$, where $e_p(x)=\Omega(|x|_p)\in L^2({\bQ}_{p})$, and Ω is a characteristic function of the unit interval $[0,\,1]$, $\bQ_p$ is the field of p-adic numbers, $p=2,3,5,...$; $\bQ_\infty=\bR$. This description allows us to construct an infinite family of Haar wavelet bases on $L^2(\bA)$ which can be obtained by shifts and multi-delations. The adelic multiresolution analysis (MRA) in $L^2(\bA)$ is also constructed. In the framework of this MRA another infinite family of Haar wavelet bases is constructed. We introduce the adelic Lizorkin spaces of test functions and distributions and give the characterization of these spaces in terms of wavelet functions. One class of pseudo-differential operators (including the fractional operator) is studied on the Lizorkin spaces. A criterion for an adelic wavelet function to be an eigenfunction for a pseudo-differential operator is derived. We prove that any wavelet function is an eigenfunction of the fractional operator. These results allow one to create the necessary prerequisites for intensive using of adelic wavelet bases and pseudo-differential operators in applications.

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