



# Constructing all self-adjoint matrices with prescribed spectrum and diagonal

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The Schur-Horn Theorem states that there exists a self-adjoint matrix with a given spectrum and diagonal if and only if the spectrum majorizes the diagonal. Though the original proof of this result was nonconstructive, several constructive proofs have subsequently been found. Most of these constructive proofs rely on Givens rotations, and none have been shown to be able to produce every example of such a matrix. We introduce a new construction method that is able to do so. This method is based on recent advances in finite frame theory which show how to construct frames whose frame operator has a given prescribed spectrum and whose vectors have given prescribed lengths. This frame construction requires one to find a sequence of eigensteps, that is, a sequence of interlacing spectra that satisfy certain trace considerations. In this paper, we show how to explicitly construct every such sequence of eigensteps. Here, the key idea is to visualize eigenstep construction as iteratively building a staircase. This visualization leads to an algorithm, dubbed Top Kill, which produces a valid sequence of eigensteps whenever it is possible to do so. We then build on Top Kill to explicitly parametrize the set of all valid eigensteps. This yields an explicit method for constructing all self-adjoint matrices with a given spectrum and diagonal, and moreover all frames whose frame operator has a given spectrum and whose elements have given lengths.

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