



Mathematics > Functional Analysis

# A $T(1)$ -Theorem for non-integral operators

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Let  $X$  be a space of homogeneous type and let  $L$  be a sectorial operator with bounded holomorphic functional calculus on  $L^2(X)$ . We assume that the semigroup  $\{e^{-tL}\}_{t>0}$  satisfies Davies-Gaffney estimates.

Associated to  $L$  are certain approximations of the identity. We call an operator  $T$  a non-integral operator if compositions involving  $T$  and these approximations satisfy certain weighted norm estimates. The Davies-Gaffney and the weighted norm estimates are together a substitute for the usual kernel estimates on  $T$  in Calderón-Zygmund theory. In this paper, we show, under the additional assumption that a vertical Littlewood-Paley-Stein square function associated to  $L$  is bounded on  $L^2(X)$ , that a non-integral operator  $T$  is bounded on  $L^2(X)$  if and only if  $T(1) \in BMO_L(X)$  and  $T^{\ast}(1) \in BMO_{L^{\ast}}(X)$ . Here,  $BMO_L(X)$  and  $BMO_{L^{\ast}}(X)$  denote the recently defined  $BMO(X)$  spaces associated to  $L$  that generalize the space  $BMO(X)$  of John and Nirenberg.

Generalizing a recent result due to F. Bernicot, we show a second version of a  $T(1)$ -Theorem under weaker off-diagonal estimates, which gives a positive answer to a question raised by him. As an application, we prove  $L^2(X)$ -boundedness of a paraproduct operator associated to  $L$ . We moreover study criterions for a  $T(b)$ -Theorem to be valid.

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