Estimates for the asymptotic behavior of the constants in the Bohnenblust--Hille inequality

G. A. Muñoz-Fernández, D. Pellegrino, J. B. Seoane-Sepúlveda

(Submitted on 24 Jul 2011)

A classical inequality due to H.F. Bohnenblust and E. Hille states that for every positive integer n there is a constant $C_{n}>0$ so that $$(\sum\limits_{i_{1},...,i_{n}=1}^{N}|U(e_{i_{^{1}}},...,e_{i_{n}})|^{\frac{2n}{n+1}})^{\frac{n+1}{2n}}\leq C_{n}||U||$$ for every positive integer N and every n-linear mapping $U:\ell_{\infty}^{N}\times...\times\ell_{\infty}^{N}\rightarrow\mathbb{C}$. The original estimates for those constants from Bohnenblust and Hille are $$C_{n}=n^{\frac{n+1}{2n}}2^{\frac{n-1}{2}}.$$ In this note we present explicit formulae for quite better constants, and calculate the asymptotic behavior of these estimates, completing recent results of the second and third authors. For example, we show that, if $C_{\mathbb{R},n}$ and $C_{\mathbb{C},n}$ denote (respectively) these estimates for the real and complex Bohnenblust--Hille inequality then, for every even positive integer n, $$\frac{C_{\mathbb{R},n}}{\sqrt{\pi}} = \frac{C_{\mathbb{C},n}}{\sqrt{2}} = 2^{\frac{n+2}{8}}\cdot r_n$$ for a certain sequence $\{r_n\}$ which we estimate numerically to belong to the interval $(1,3/2)$ (the case n odd is similar). Simultaneously, assuming that $\{r_n\}$ is in fact convergent, we also conclude that $$\displaystyle \lim_{n \rightarrow \infty} \frac{C_{\mathbb{R},n}}{C_{\mathbb{R},n-1}} = \displaystyle \lim_{n \rightarrow \infty} \frac{C_{\mathbb{C},n}}{C_{\mathbb{C},n-1}}= 2^{1/8}.$$

Comments:	7 pages
Subjects:	**Functional Analysis (math.FA)**
Journal reference:	Linear and Multilinear Algebra 60 (2012) 573-582
Cite as:	**arXiv:1107.4814 [math.FA]**
	(or **arXiv:1107.4814v1 [math.FA]** for this version)

Submission history

From: Daniel M. Pellegrino [view email]
[v1] Sun, 24 Jul 2011 23:52:30 GMT (8kb)