



A generalization of Marstrand's theorem for projections of cartesian products

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We prove the following variant of Marstrand's theorem about projections of cartesian products of sets:

Let K_1, \dots, K_n Borel subsets of $\mathbb{R}^{m_1}, \dots, \mathbb{R}^{m_n}$ respectively, and $\pi: \mathbb{R}^{m_1} \times \dots \times \mathbb{R}^{m_n} \rightarrow \mathbb{R}^k$ be a surjective linear map. We set $\mathfrak{m} = \min\{\sum_{i \in I} \dim_H(K_i) + \dim \pi(\bigoplus_{i \in I^c} \mathbb{R}^{m_i})\}$, $I \subset \{1, \dots, n\}$, $I \neq \emptyset$. Consider the space $\Lambda_m = \{(t, O), t \in \mathbb{R}, O \in SO(m)\}$ with the natural measure and set $\Lambda = \Lambda_{m_1} \times \dots \times \Lambda_{m_n}$. For every $\lambda = (t_1, O_1, \dots, t_n, O_n) \in \Lambda$ and every $x = (x^1, \dots, x^n) \in \mathbb{R}^{m_1} \times \dots \times \mathbb{R}^{m_n}$ we define $\pi_\lambda(x) = \pi(t_1 O_1 x^1, \dots, t_n O_n x^n)$. Then we have (i) If $\mathfrak{m} > k$, then $\pi_\lambda(K_1 \times \dots \times K_n)$ has positive k -dimensional Lebesgue measure for almost every $\lambda \in \Lambda$. (ii) If $\mathfrak{m} \leq k$ and $\dim_H(K_1 \times \dots \times K_n) = \dim_H(K_1) + \dots + \dim_H(K_n)$, then $\dim_H(\pi_\lambda(K_1 \times \dots \times K_n)) = \mathfrak{m}$ for almost every $\lambda \in \Lambda$.

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