Mathematics > Classical Analysis and ODEs

A generalization of Marstrand's theorem for projections of cartesian products

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We prove the following variant of Marstrand's theorem about projections of cartesian products of sets:
Let $K_1,...,K_n$ Borel subsets of $\mathbb R^{m_1},... ,\mathbb R^{m_n}$ respectively, and $\pi:\mathbb R^{m_1}\times...\times\mathbb R^{m_n} \to\mathbb R^k$ be a surjective linear map. We set $$\mathfrak {m}:=\min\{\sum_{i\in I}\dim_H(K_i) + \dim\pi(\bigoplus_{i\in I^c}\mathbb R^ {m_i}), I\subset\{1,...,n\}, I\ne\emptyset\}.$$ Consider the space $\Lambda_m=\{(t,O), t\in\mathbb R, O\in SO(m)\}$ with the natural measure and set $\Lambda=\Lambda_{m_1}\times...\times\Lambda_{m_n}$. For every $\lambda=(t_1,O_1,...,t_n,O_n)\in\Lambda$ and every $x=(x^1,,x^n)\in\mathbb R^{m_1}\times...\times\mathbb R^{m_n}$ we define $\pi_\lambda(x)=\pi (t_1O_1x^1,...,t_nO_nx^n)$. Then we have (i) If $\mathfrak{m}>k$, then $\pi_\lambda(K_1\times...\times K_n)$ has positive k-dimensional Lebesgue measure for almost every $\lambda\in\Lambda$. (ii) If $\mathfrak{m}\leq k$ and $\dim_H(K_1\times...\times K_n)=\dim_H(K_1)+...+\dim_H(K_n)$, then \dim_H(\pi_\lambda(K_1\times...\times K_n))=\mathfrak{m}$ for almost every $\lambda\in\Lambda$.

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