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Nuclearity Related Properties in Operator Systems

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Some recent research on the tensor products of operator systems and ensuing nuclearity properties in this setting raised many stability problems. In this paper we examine the preservation of these nuclearity properties including exactness, local liftability and the double commutant expectation property under basic algebraic operations such as quotient, duality, coproducts and tensor products. We show that, in the finite dimensional case, exactness and lifting property are dual pairs, that is, an operator system S is exact if and only if the dual operator system S^d has the lifting property. Moreover, the lifting property is preserved under quotients by null subspaces. Again in the finite dimensional case we prove that every operator system has the k-lifting property in the sense that whenever $f:S -> A/I$ is a unital and completely positive map, where A is a C*-algebra and I is an ideal, then f possess a unital k-positive lift on A, for every k. This property provides a novel proof of a classical result of Smith and Ward on the preservation of matricial numerical ranges of an operator.

The Kirchberg conjecture naturally falls into this context. We show that the Kirchberg conjecture is equivalent to the statement that the five dimensional universal operator system generated by two contraction (S_2) has the double commutant expectation property. In addition to this we give several equivalent statements to this conjecture regarding the preservation of various nuclearity properties under basic algebraic operations. We show that the Smith Ward problem is equivalent to the statement that every three dimensional operator system has the lifting property (or exactness). If we suppose that both the Kirchberg conjecture and the Smith Ward problem have an affirmative answer then this implies that every three dimensional operator system is C*-nuclear.

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