Mathematics > Operator Algebras

Square functions for Ritt operators on noncommutative L^p-spaces

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For any Ritt operator T acting on a noncommutative L^p-space, we define the notion of \textit{completely} bounded functional calculus $H^\infty (B_\gamma)$ where B_γ is a Stolz domain. Moreover, we introduce the `column square functions' $\norm{x}_{T,c,\alpha}=\Bnorm{\Big(\sum_{k=1} ^{+\infty}k^{2\alpha-1}|T^{k-1}(I-T)^{\alpha}(x)|^2\Big)^{1/2}}_{L^p(M)}$ and the `row square functions' $\norm{x}_{T,r,\alpha}=\Bnorm{\Big(\sum_{k=1}^ {+\infty}k^{2\alpha-1} |\Big(T^{k-1}(I-T)^{\alpha}(x)\Big)^*|^2\Big)^{1/2}}_{L^p(M)} $ for any $\alpha>0$ and any $x\in L^p(M)$. Then, we provide an example of Ritt operator which admits a completely bounded $H^\infty(B_\gamma)$ functional calculus for some $\gamma \in \big]0,\frac{\pi}{2}\big[$ such that the square functions $\norm{\cdot}_{T,c,\alpha}$ and $\norm{\cdot}_{T,r,\alpha}$ are not equivalent. Moreover, assuming $1<p<2$ and $\alpha>0$, we prove that if $\Ran (I-T)$ is dense and T admits a completely bounded $H^\infty (B_\gamma)$ functional calculus for some $\gamma \in \big]0,\frac{\pi}{2}\big[$ then there exists a positive constant C such that for any $x \in L^p(M)$, there exists $x_1, x_2 \in L^p(M)$ satisfying $x=x_1+x_2$ and $\norm{x_1}_ {T,c,\alpha}+\norm{x_2}_{T,r,\alpha}\leq C \norm{x}_{L^p(M)}$. Finally, we observe that this result applies to a suitable class of selfadjoint Markov maps on noncommutative L^p-spaces.

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