An interior-point trust-region polynomial algorithm for convex programming

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Abstract. An interior-point trust-region algorithm is proposed for minimization of a convex quadratic objective function over a general convex set. The algorithm uses a trust-region model to ensure descent on a suitable merit function. The complexity of our algorithm is proved to be as good as the interior-point polynomial algorithm.

Key words. interior-point algorithm, self-concordant barrier, trust-region subproblem.

1 Introduction

The idea of interior-point trust-region algorithm can be traced back to Dikin(1967) where an interior ellipsoid method was developed for linear problems. Recently, Tseng(2004)produced a global and local convergence analysis of Dikin's algorithm for indefinite quadratic programming. We also refer Absil and Tits(2005) for this direction. Ye(1992) developed an affine scaling algorithm for indefinite quadratic programming by solving sequential trust-region subproblem. Global first-order and second-order convergence results were proved, and later enhanced by Sun(1993) for convex case. An affine-scaling potential-reduction interior-point trust-region algorithm was developed for the indefinite quadratic programming in the chapter 9 of Ye(1997). This algorithm has recently been extended to solve symmetric cone programming by Faybusovich and Lu(2004). In the trust-region literature, Conn, Gould and Toint (2000) developed a primal barrier trust-region algorithm, which has been recently extended to solve symmetric cone

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programming by Lu and Yuan(2005). In this paper, we present an affine-scaling primal barrier interior-point trust-region algorithm. Our algorithm minimizes a convex quadratic objective function over a general convex set, which is the first interior-point trust-region algorithm of covering basically the whole convex programming! Moreover, by using the techniques and properties in both interior-point algorithms and trust-region methods literature, we show that the complexity of our algorithm is as good as the interior-point polynomial algorithms! This gives us the strong theoretical support for the good practical performance of the interior-point trust-region algorithm in Lu and Yuan(2005). Although our analysis is based on a fixed trust-region radius and solving the trust-region subproblem exactly in each step of our algorithm, the frame of the interior-point trust-region algorithm allows us to make the trust-region radius flexible and use iterative methods to solve the trust-region subproblem approximately in practical implementation. This advantage makes the interior-point trust-region algorithm competitive with the pure interior-point algorithm for solving large-scale problems. The goal of this paper is to show that the complexity of interior-point trust-region algorithm is as good as the complexity of pure interior-point algorithm in convex programming.

2 Self-concordant barrier and its properties

In this section, we present the concept of self-concordant barrier and its properties that will play an important in our analysis of section 3.

We assume that K is a convex set in a finite-dimensional real vector space E. The following is the definition of self-concordant barrier, which is due to Nesterov and Nemirovskii(1994).

Definition 2.1. *Let $F : K^\circ \to R$ be a C^3-smooth convex function such that $F(x) \to \infty$ as $x \in K^\circ$ approaches the boundary of K and*

$$| F^{'''}(x)[h,h,h] | \leq 2 \langle F^{''}(x)h, h\rangle^{3/2} \tag{2.1}$$

for all $x \in K^\circ$ and for all $h \in E$. Then F is called a self-concordant function for K. F is called a self-concordant barrier if F is a self-concordant function and

$$\vartheta := sup_{x\in K^\circ} \langle F'(x), F''(x)^{-1}F'(x)\rangle < \infty. \tag{2.2}$$

ϑ is called barrier parameter of F.

Let $F''(x)$ denote the Hessian of a self-concordant function $F(x)$. Since it is positive definite, for every $x \in K^\circ$, $\|v\|_x = \langle v, F''(x)v \rangle^{\frac{1}{2}}$ is a norm on E induced by $F''(x)$. Let $B_x(y, r)$ denote the open ball of radius r centered at y, where the radius is measured w.r.t. $\| \; \|_x$. This ball is called the Dikin ball. The following lemmas are very crucial for the analysis of our algorithm in the next section. For the proofs, see e.g. the chapter 2 of Renegar(2001).

Lemma 2.1. *Assume $F(x)$ is a self-concordant function for K, then for all $x \in K^\circ$, we have $B_x(x, 1) \subseteq K^\circ$ and if whenever $y \in B_x(x, 1)$ we have*

$$\frac{\|v\|_y}{\|x\|_x} \leq \frac{1}{1 - \|y - x\|_x} \; for \; all \; v \neq 0. \tag{2.3}$$

Lemma 2.2. *Assume $F(x)$ is a self-concordant function for K, $x \in K^\circ$ and $y \in B_x(x, 1)$, then*

$$|F(y) - F(x) - \langle F'(x), y - x \rangle - \frac{\langle y - x, F''(x)(y - x) \rangle}{2}| \leq \frac{\|y - x\|_x^3}{3(1 - \|y - x\|_x)}. \tag{2.4}$$

Let $n(x) := -F''(x)^{-1}F'(x)$ be the Newton step of $F(x)$.

Lemma 2.3. *Assume $F(x)$ is a self-concordant function. If $\|n(x)\|_x \leq \frac{1}{4}$ then $F(x)$ has a minimizer z and*

$$\|z - x\|_x \leq \|n(x)\|_x + \frac{3\|n(x)\|_x^2}{(1 - \|n(x)\|_x)^3}. \tag{2.5}$$

Lemma 2.4. *Assume $F(x)$ is a self-concordant barrier with barrier parameter ϑ. If $x, y \in K^\circ$ then*

$$\langle F'(x), y - x \rangle \leq \vartheta. \tag{2.6}$$

3 The interior-point trust-region algorithm

In this section, we present our algorithm and give the complexity analysis.

We consider the following optimization problem

$$\min \qquad q(x) = \frac{1}{2}\langle x, Qx \rangle + \langle c, x \rangle \tag{3.1}$$

$$\text{subject to} \qquad x \in K. \tag{3.2}$$

Here $Q : E \mapsto E$ is a positive definite or positive semi-definite linear operator, $c \in E$. K is a bounded convex set with nonempty relative interior.

We assume $F(x)$ is the self-concordant barrier for the symmetric cone K and define the merit function as

$$f_{\eta_k}(x) = \eta_k q(x) + F(x). \tag{3.3}$$

We should mention here that $f_{\eta_k}(x)$ is a self-concordant function by the definition (2.1)! We want to decrease the value of $f_{\eta_k}(x)$ for every fixed η_k in each inner iteration, and increase η_k to positive infinity in outer iterations. From Lemma 2.1, for any $x_{k,j} \in K^\circ$ and $d \in E$, we have that $x_{k,j} + d \in K^\circ$ provided that $\|F''(x_{k,j})^{\frac{1}{2}} d\| \le \alpha_{k,j} < 1$. It follows from Lemma 2.2 that

$$\begin{aligned} F(x_{k,j}+d) - F(x_{k,j}) &\le \langle F'(x_{k,j}), d\rangle + \frac{\langle d, F''(x_{k,j})d\rangle}{2} + \frac{\|d\|^3_{x_{k,j}}}{3(1-\|d\|_{x_{k,j}})} \\ &\le \langle F'(x_{k,j}), d\rangle + \frac{\langle d, F''(x_{k,j})d\rangle}{2} + \frac{\alpha^3_{k,j}}{3(1-\alpha_{k,j})} \end{aligned} \tag{3.4}$$

Therefore, we get

$$\begin{aligned} f_{\eta_k}(x_{k,j}+d) \quad - \quad & f_{\eta_k}(x_{k,j}) \le \frac{\langle d, (\eta_k Q + F''(x_{k,j}))d\rangle}{2} \\ & + \langle \eta_k(Qx_{k,j}+c) + F'(x_{k,j}), d\rangle + \frac{\alpha^3_{k,j}}{3(1-\alpha_{k,j})}. \end{aligned} \tag{3.5}$$

Now, it is obviously that for decreasing $f_{\eta_k}(x)$, we solve the following trust-region subproblem

$$\min \quad \tfrac{1}{2}\langle d, (\eta_k Q + F''(x_{k,j}))d\rangle + \langle \eta_k(Qx_{k,j}+c) + F'(x_{k,j}), d\rangle = m_{k,j}(d) \tag{3.6}$$

$$s.t. \quad \|F''(x_{k,j})^{\frac{1}{2}} d\|^2 \le \alpha^2_{k,j}. \tag{3.7}$$

Define

$$Q_{k,j} = \eta_k F''(x_{k,j})^{-\frac{1}{2}} Q F''(x_{k,j})^{-\frac{1}{2}} + I, \tag{3.8}$$

$$c_{k,j} = F''(x_{k,j})^{-\frac{1}{2}} \left(\eta_k(Qx_{k,j}+c) + F'(x_{k,j})\right), \tag{3.9}$$

and using the transformation

$$d' = F''(x_{k,j})^{\frac{1}{2}} d, \tag{3.10}$$

the above problem (3.6)-(3.7) can be rewritten as

$$\min \quad q'_{k,j}(d') = \tfrac{1}{2}\langle d', Q_{k,j} d'\rangle + \langle c_{k,j}, d'\rangle \tag{3.11}$$

$$\|d'\|^2 \leq \alpha_{k,j}^2. \tag{3.12}$$

Once $d'_{k,j}$ is computed, we obtain the step

$$d_{k,j} = F''(x_{k,j})^{-\frac{1}{2}} d'_{k,j}, \tag{3.13}$$

and it follows from inequality (3.5) that

$$f_{\eta_k}(x_{k,j} + d_{k,j}) \quad - \quad f_{\eta_k}(x_{k,j}) \leq q'_{k,j}(d'_{k,j}) + \frac{\alpha_{k,j}^3}{3(1-\alpha_{k,j})}. \tag{3.14}$$

Algorithm 3.1. *(An Interior-Point Trust Region Algorithm)*

Step 0 **Initialization.** *An initial point* $x_{0,0} \in K^\circ$ *and an initial parameter* $\eta_0 > 0$ *are given. Set* $\alpha_{k,j} = \alpha < 1$ *for some constant* α*. Set* $k = 0$ *and* $j = 0$*.*

Step 1 **Test inner iteration termination.** *If*

$$\langle c_{k,j}, Q_{k,j}^{-1} c_{k,j}\rangle \leq \frac{1}{9}, \tag{3.15}$$

set $x_{k+1,0} = x_{k,j}$ *and go to Step 3.*

Step 2 **Step calculation.** *Solve problem (3.11)-(3.12) obtaining* $d'_{k,j}$ *exactly, set* $d_{k,j}$ *by (3.13) and* $x_{k,j+1} = x_{k,j} + d_{k,j}$*.*

Step 3 **Update parameter** η**.** *Set* $\eta_{k+1} = \theta\eta_k$ *for some constant* $\theta > 1$*. Increase* k *by 1 and go to step 1.*

Theorem 3.1. *a)If we choose* $\alpha = \frac{1}{4}$*, then*

$$f_{\eta_k}(x_{k,j+1}) - f_{\eta_k}(x_{k,j}) < -\frac{1}{48}, \tag{3.16}$$

which is independent of k *and* j*.*
b)If the initial point $x_{0,0}$ *satisfies the condition (3.15), then for any* $\epsilon > 0$*, we will get a solution* x *such that* $q(x) - q(x^*) < \epsilon$ *in at most*

$$\frac{48\theta(\vartheta + \sqrt{\vartheta}) \ln \frac{\vartheta + \sqrt{\vartheta}}{\epsilon\eta_0}}{\ln \theta} \tag{3.17}$$

steps, here $x^* = argmin_{x \in K} q(x)$*.*

To prove part a) of this theorem, we need the following lemma which is well-known in trust-region literature.

Lemma 3.1. *Any global minimizer $d'_{k,j}$ of problem (3.11)-(3.12) satisfies the equation*

$$(Q_{k,j} + \mu_{k,j} I) d'_{k,j} = -c_{k,j}, \tag{3.18}$$

here $Q_{k,j} + \mu_{k,j} I$ is positive semi-definite, $\mu_{k,j} \geq 0$ and $\mu_{k,j}(\|d'_{k,j}\| - \alpha_{k,j}) = 0$.

For a proof, see e.g. Section 7.2 of Conn, Gould and Toint (2000).

Proof of Theorem 3.1 part a). If the solution of (3.11)-(3.12) lies on the boundary of the trust-region, that is, $\|d'_{k,j}\| = \alpha_{k,j}$, then

$$\begin{aligned} q'_{k,j}(d'_{k,j}) &= \frac{1}{2}\langle d'_{k,j}, Q_{k,j} d'_{k,j}\rangle + \langle c_{k,j}, d'_{k,j}\rangle \\ &= \langle d'_{k,j}, Q_{k,j} d'_{k,j} + c_{k,j}\rangle - \frac{1}{2}\langle d'_{k,j}, Q_{k,j} d'_{k,j}\rangle \\ &= \langle d'_{k,j}, -\mu_{k,j} d'_{k,j}\rangle - \frac{1}{2}\langle d'_{k,j}, (\eta_k F''(x_{k,j})^{-\frac{1}{2}} Q F''(x_{k,j})^{-\frac{1}{2}} + I) d'_{k,j}\rangle \\ &= -\mu_{k,j}\alpha_{k,j}^2 - \frac{1}{2}\langle d'_{k,j}, \eta_k F''(x_{k,j})^{-\frac{1}{2}} Q F''(x_{k,j})^{-\frac{1}{2}} d'_{k,j}\rangle - \frac{1}{2}\alpha_{k,j}^2 \\ &\leq -\frac{1}{2}\alpha_{k,j}^2 = -\frac{1}{32}, \end{aligned}$$

here the third equality follows from the equalities (3.8) and (3.18), and the inequality follows from the fact that $\eta_k F''(x_{k,j})^{-\frac{1}{2}} Q F''(x_{k,j})^{-\frac{1}{2}}$ is positive definite or positive semi-definite. Therefore, it follows from inequality (3.14) that

$$f_{\eta_k}(x_{k,j+1}) - f_{\eta_k}(x_{k,j}) \leq -\frac{1}{32} + \frac{(\frac{1}{4})^3}{3(1-\frac{1}{4})} < -\frac{1}{48}.$$

If the solution of (3.11)-(3.12) lies in the interior of the trust-region, that is, $\|d'_{k,j}\| < \alpha_{k,j}$, then from Lemma 3.1 we know $\mu_{k,j} = 0$ and consequently $d'_{k,j} = -Q_{k,j}^{-1} c_{k,j}$, and

$$q'_{k,j}(d'_{k,j}) = \frac{1}{2}\langle d'_{k,j}, Q_{k,j} d'_{k,j}\rangle + \langle c_{k,j}, d'_{k,j}\rangle = -\frac{1}{2}\langle c_{k,j}, Q_{k,j}^{-1} c_{k,j}\rangle. \tag{3.20}$$

By the mechanism of our algorithm, we know that $\langle c_{k,j}, Q_{k,j}^{-1} c_{k,j}\rangle > \frac{1}{9}$ for all k and j. Therefore,

$$f_{\eta_k}(x_{k,j+1}) - f_{\eta_k}(x_{k,j}) \leq -\frac{1}{18} + \frac{(\frac{1}{4})^3}{3(1-\frac{1}{4})} < -\frac{1}{48}.$$

Therefore, we complete our proof. $\square$

Let $n_{\eta_k}(x_{k,j})$ be the Newton step of $f_{\eta_k}(x)$ at the point $x_{k,j}$. We should point out that

$$\begin{aligned}\|n_{\eta_k}(x_{k,j})\|_{x_{k,j}} &= \langle f'_{\eta_k}(x_{k,j}), f''_{\eta_k}(x_{k,j})^{-1} f'_{\eta_k}(x_{k,j})\rangle \\ &= \langle \eta_k(Qx_{k,j}+c)+F'(x_{k,j}), (\eta_k Q + F''(x_{k,j}))^{-1}(\eta_k(Qx_{k,j}+c)+F'(x_{k,j}))\rangle \\ &= \langle c_{k,j}, Q_{k,j}^{-1} c_{k,j}\rangle,\end{aligned}$$

where last equality follows equalities (3.8) and (3.9). This equality connects equality (3.19) and the assumption of the following two lemmas, which tells us that we can stop the inner iteration if the reduction of the objective function with an interior solution is smaller than some constant. The following two lemmas are extension of Renegar(2001)'s result of minimizing a linear objective function over a convex set.

Lemma 3.2. *Let $x^* = argmin_{x\in K} q(x)$. If $\|n_\eta(x)\|_x \le \frac{1}{9}$, then*

$$q(x) - q(x^*) \le \frac{\vartheta + \sqrt{\vartheta}}{\eta}. \tag{3.22}$$

Proof. Let $x(\eta) = argmin_{x\in K} f_\eta(x)$. Then

$$\begin{aligned} q(x(\eta)) - q(x^*) &\le \langle q'(x(\eta)), x(\eta) - x^*\rangle \\ &= \langle \frac{-F'(x(\eta))}{\eta}, x(\eta) - x^*\rangle \le \frac{\vartheta}{\eta} \end{aligned} \tag{3.23}$$

The first inequality is by the convexity of $q(x)$. The equality follows from the fact that $f'(x(\eta)) = 0$, and the last inequality follows from Lemma 2.4.

It easily follows from Lemma 2.3 that

$$\|x - x(\eta)\|_x \le \frac{1}{9} + \frac{3(\frac{1}{9})^2}{(1-\frac{1}{9})^3} < \frac{1}{4} \tag{3.24}$$

and consequently from Lemma 2.1 that

$$\|x - x(\eta)\|_{x(\eta)} \le \frac{\|x - x(\eta)\|_x}{1 - \|x - x(\eta)\|_x} < \frac{1}{3}. \tag{3.25}$$

Then we have

$$
\begin{aligned}
q(x) - q(x(\eta)) &= \langle q'(x(\eta)), x - x(\eta)\rangle + \frac{\langle x - x(\eta), Q(x - x(\eta))\rangle}{2} \\
&= \langle \frac{-F'(x(\eta))}{\eta}, x - x(\eta)\rangle + \frac{\langle x - x(\eta), \eta Q(x - x(\eta))\rangle}{2\eta} \\
&\leq \frac{\langle -F''(x(\eta))^{-\frac{1}{2}}F'(x(\eta)), F''(x(\eta))^{\frac{1}{2}}(x - x(\eta))\rangle}{\eta} + \frac{\|x - x(\eta)\|_x}{2\eta} \\
&\leq \frac{\|F''(x(\eta))^{-\frac{1}{2}}F'(x(\eta))\|\|F''(x(\eta))^{\frac{1}{2}}(x - x(\eta))\|}{\eta} + \frac{1}{8\eta} \\
&\leq \frac{\sqrt{\vartheta}\|x - x(\eta)\|_{x(\eta)}}{\eta} + \frac{1}{8\eta} \leq \frac{\sqrt{\vartheta}}{3\eta} + \frac{1}{8\eta} \leq \frac{\sqrt{\vartheta}}{\eta}, \qquad (3.26)
\end{aligned}
$$

where the last inequality follows from the fact that ϑ is always greater than 1 and the third last inequality follows from the definition of ϑ. By adding the inequality (3.21) and (3.24), we get inequality (3.20). □

This lemma tells us that to get an ϵ-solution, we only need

$$
\eta_k = \eta_0 \theta^k \geq \frac{\vartheta + \sqrt{\vartheta}}{\epsilon},
$$

and consequently

$$
k \geq \frac{\ln \frac{\vartheta + \sqrt{\vartheta}}{\epsilon \eta_0}}{\ln \theta} \qquad (3.28)
$$

outer iterations.

Lemma 3.3. *If* $\|n_{\eta_k}(x)\|_x \leq \frac{1}{9}$*, then*

$$
f_{\eta_{k+1}}(x) - f_{\eta_{k+1}}(x(\eta_{k+1})) \leq \theta(\vartheta + \sqrt{\vartheta}). \qquad (3.29)
$$

Proof. First, we have

$$
\begin{aligned}
f_{\eta_{k+1}}(x) - f_{\eta_{k+1}}(x(\eta_k)) &\leq \langle f'_{\eta_{k+1}}(x), x - x(\eta_k)\rangle = \langle \eta_{k+1}(Qx+c) + F'(x), x - x(\eta_k)\rangle \\
&= \frac{\eta_{k+1}}{\eta_k}\langle \eta_k(Qx+c) + F'(x), x - x(\eta_k)\rangle + (\frac{\eta_{k+1}}{\eta_k} - 1)\langle F'(x), x(\eta_k) - x\rangle \\
&= \theta\langle f''^{-\frac{1}{2}}_{\eta_k}(x)(\eta_k(Qx+c) + F'(x)), f''_{\eta_k}(x)^{\frac{1}{2}}(x - x(\eta_k))\rangle \\
&\quad +(\theta - 1)\langle F''(x)^{-\frac{1}{2}}F'(x), F''(x)^{\frac{1}{2}}(x(\eta_k) - x)\rangle \\
&\leq \theta\| f''^{-\frac{1}{2}}_{\eta_k}(x)(\eta_k(Qx+c) + F'(x))\|\| f''_{\eta_k}(x)^{\frac{1}{2}}(x - x(\eta_k))\| \\
&\quad +(\theta - 1)\|F''(x)^{-\frac{1}{2}}F'(x)\|\|F''(x)^{\frac{1}{2}}(x(\eta_k) - x)\| \\
&\leq \theta\|n_{\eta_k}(x)\|_x\|x(\eta_k) - x\|_x + (\theta - 1)\sqrt{\vartheta}\|x(\eta_k) - x\|_x \\
&\leq \theta\frac{1}{9}\frac{1}{4} + (\theta - 1)\sqrt{\vartheta}\frac{1}{4} \leq \theta\sqrt{\vartheta}, \qquad (3.30)
\end{aligned}
$$

where the first inequality follows from the convexity of $f_{\eta_{k+1}}(x)$ and the second last inequality follows from inequality (3.22).

Similarly, we have

$$
\begin{aligned}
f_{\eta_{k+1}}(x(\eta_k)) - f_{\eta_{k+1}}(x(\eta_{k+1})) &\leq \langle f'_{\eta_{k+1}}(x(\eta_k)), x(\eta_k) - x(\eta_{k+1})\rangle \\
&= \langle \eta_{k+1}(Qx(\eta_k) + c) + F'(x(\eta_k)), x(\eta_k) - x(\eta_{k+1})\rangle \\
&= \frac{\eta_{k+1}}{\eta_k}\langle \eta_k(Qx(\eta_k) + c) + F'(x(\eta_k)), x(\eta_k) - x(\eta_{k+1})\rangle \\
&\quad +(\frac{\eta_{k+1}}{\eta_k} - 1)\langle F'(x(\eta_k)), x(\eta_{k+1}) - x(\eta_k)\rangle \\
&= \theta\langle f'_{\eta_k}(x(\eta_k)), x(\eta_k) - x(\eta_{k+1})\rangle \\
&\quad +(\theta - 1)\langle F'(x(\eta_k)), x(\eta_{k+1}) - x(\eta_k)\rangle \\
&= (\theta - 1)\langle F'(x(\eta_k)), x(\eta_{k+1}) - x(\eta_k)\rangle < \theta\vartheta, \qquad (3.31)
\end{aligned}
$$

where the last inequality follows from Lemma 2.4 and the last equality holds because $x(\eta_k)$ minimizes $f_{\eta_k}(x)$ and hence $f'_{\eta_k}(x(\eta_k)) = 0$. By adding inequality (3.27) and inequality (3.28), we get inequality (3.26). □

This lemma and Theorem (3.1) part a) tell us that we need at most

$$48(\vartheta + \sqrt{\vartheta}) \qquad (3.32)$$

steps in each inner iteration.

Proof of Theorem 3.1 part b). It follows from (3.25) and (3.29). □

We want to mention that even the initial point doesn't satisfy condition (3.15), our algorithm will make it satisfy in polynomial-time, which follows from the part a) of Theorem 3.1.

4 Concluding remarks

In this paper, we have shown that the complexity of interior-point trust-region algorithm is as good as pure interior-point algorithm in convex programming. This gives us a bridge of connecting interior-point methods and trust-region methods. It is our belief that more efficient algorithm can be built by taking advantages of both methods. The bottom line is that we won't lose any complexity priority by the result of this paper.

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