计算数学
MATHEMATICA NUMERICA SINICA
主办:中国科学院数学与系统科学研究院
ISSN 0254-7791 CN 11-2125/O1

计算数学 » 2011, Vol. 33 » Issue (4): 397-408 DOI:

非匹配网格上Stokes-Darcy模型的非协调元方法及其预条件技术

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NONCONFORMING ELEMENT METHODS AND PRECONDITIONING TECHNIQUES FOR STOKES-DARCY MODEL ON NONMATCHING GRIDS

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摘要 本文讨论了非匹配网格上Stokes-Darcy模型的两种低阶非协调元方法, 证明了离散问题的适定性并得到了最优的误差估计. 对离散出来的非对称不定线性方程组, 我们提出了几种有效的预条件子, 证明了预条件子的最优性.最后, 数值试验验证了我们的理论结果.

关键词: 非协调元 非匹配网格 Stokes-Darcy模型 预条件子

Abstract: In this paper, two lower order nonconforming finite element methods are presented for the Stokes-Darcy model on nonmatching grids, the well-posedness of the discrete problem is proved and the optimal error estimate is also derived. Moreover, we propose some efficient preconditioners for the nonsymmetric and indefinite linear system of the algebraic equations, and prove the optimality of our preconditioners. Finally, numerical experiments are given to confirm our theoretical results.

Key words: nonconforming element nonmatching grids Stokes-Darcy model preconditioner

收稿日期: 2011-05-15;

基金资助:

国家自然科学基金(项目号11071124和10871100)资助.

引用本文:

. 非匹配网格上Stokes-Darcy模型的非协调元方法及其预条件技术[J]. 计算数学, 2011, 33(4): 397-408.

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