Numerical Bifurcation Analysis of Conformal Formulations of the Einstein Constraints

M. Holst, V. Kungurtsev

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The Einstein constraint equations have been the subject of study for more than fifty years. The introduction of the conformal method in the 1970's as a parameterization of initial data for the Einstein equations led to increased interest in the development of a complete solution theory for the constraints, with the theory for constant mean curvature (CMC) spatial slices and closed manifolds completely developed by 1995. The first general non-CMC existence result was establish by Holst et al. in 2008, with extensions to rough data by Holst et al. in 2009, and to vacuum spacetimes by Maxwell in 2009. The non-CMC theory remains mostly open; moreover, recent work of Maxwell on specific symmetry models sheds light on fundamental non-uniqueness problems with the conformal method as a parameterization in non-CMC settings. In parallel with these mathematical developments, computational physicists have uncovered surprising behavior in numerical solutions to the extended conformal thin sandwich formulation of the Einstein constraints. In particular, numerical evidence suggests the existence of multiple solutions with a quadratic fold, and a recent analysis of a simplified model supports this conclusion. In this article, we examine this apparent bifurcation phenomena in a methodical way, using modern techniques in bifurcation theory and in numerical homotopy methods. We first review the evidence for the presence of bifurcation in the Hamiltonian constraint in the time-symmetric case. We give a brief introduction to the mathematical framework for analyzing bifurcation phenomena, and then develop the main ideas behind the construction of numerical homotopy, or path-following, methods in the analysis of bifurcation phenomena. We then apply the continuation software package AUTO to this problem, and verify the presence of the fold with homotopy-based numerical methods.

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