

Mathematical Physics

Counterexamples to Ferromagnetic Ordering of Energy Levels

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The Heisenberg ferromagnet has symmetry group $\mathrm{SU}(2)$. The property known as ferromagnetic ordering of energy levels (FOEL) states that the minimum energy eigenvalue among eigenvectors with total spin S is monotone decreasing as a function of S . While this property holds for certain graphs such as open chains, in this note we demonstrate some counterexamples. We consider the spin $1/2$ model on rings of length $2n$ for $n=2,3,\dots,8$, and show that the minimum energy among all spin singlets is less than or equal to the minimum energy among all spin triplets, which violates FOEL. This also shows some counterexamples to the "Aldous ordering" for the symmetric exclusion process. We also review some of the literature related to these examples.

Comments: We corrected an earlier misinterpretation we made of a famous result of Sutherland, which an anonymous referee corrected us on. 29 pages

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